Charge dependence of elliptic flow in heavy-ion collisions

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PRD92 (2015) 114010
NPA947 (2016) 155
Outline

• Recent STAR measurement of $\Delta v_2^\pi = v_2^{\pi^-} - v_2^{\pi^+}$
• Computing $v_n^\pm$ for anisotropically deformed Gubser flow
• An attempt to explain the STAR result
Observation of Charge Asymmetry Dependence of Pion Elliptic Flow and the Possible Chiral Magnetic Wave in Heavy-Ion Collisions

(STAR Collaboration)

We present measurements of $\pi^-$ and $\pi^+$ elliptic flow, $v_2$, at midrapidity in Au + Au collisions at $\sqrt{s_{NN}} = 200$, 62.4, 39, 27, 19.6, 11.5, and 7.7 GeV, as a function of event-by-event charge asymmetry, $A_{ch}$, based on data from the STAR experiment at RHIC. We find that $\pi^-$ ($\pi^+$) elliptic flow linearly increases (decreases) with charge asymmetry for most centrality bins at $\sqrt{s_{NN}} = 27$ GeV and higher. At $\sqrt{s_{NN}} = 200$ GeV, the slope of the difference of $v_2$ between $\pi^-$ and $\pi^+$ as a function of $A_{ch}$ exhibits a centrality dependence, which is qualitatively similar to calculations that incorporate a chiral magnetic wave effect. Similar centrality dependence is also observed at lower energies.

$$v_2^{\pi^-} - v_2^{\pi^+} \approx r A_{ch}$$

$$A_{ch} = \frac{N_+ - N_-}{N_+ + N_-}$$
Chiral magnetic wave (CMW)

Burnier, Kharzeev, Liao, Yee, PRL 107 (2011) 052303

Strong magnetic field and chiral anomaly induce a quadrupole deformation of electric charges.

Predicted relation

\[ v_2^{X^-} - v_2^{X^+} \approx r A_{ch} \]

The slope parameter \( r \) is positive for all hadron species.

Has a characteristic peak in mid-central collisions

- STAR data consistent with CMW interpretation
- Is this the only explanation? Let’s see what ‘normal’ hydrodynamics has to say.
\[ \Delta v_2^\pi \text{ in `normal' hydro} \]

Cooper-Frye formula

\[ (2\pi)^3 \frac{dN^\pm}{dY \rho_T dp_T d\phi_p} = \int_\Sigma (-p^\mu d\sigma_\mu) \exp \left( \frac{u \cdot p^\pm \mu_I}{T} \right) \]

\[ \propto 1 + 2v_n^\pm (p_T) \cos n\phi_p \]

In ideal hydro and for the conformal equation of state, \( \mu_I / T \) is a constant.

\[ \Delta v_2^\pi = 0 \]

To get nonvanishing \( \Delta v_2^\pi \),

1. Include viscosity \( \rightarrow \) This talk
2. Include quantum statistics \( \rightarrow \) Hongo, Hirono, Hirano, 1309.2823
3. ....
Hydrodynamic equation with conserved charges

\[ \nabla_\mu T^{\mu \nu} = 0 \quad \nabla_\mu J^\mu = 0 \]

\[ T^{\mu \nu} = \varepsilon u^\mu u^\nu + p(u^\mu u^\nu + g^{\mu \nu}) - 2\eta \sigma^{\mu \nu} \]

\[ J^\mu = n u^\mu - \kappa (u^\mu u^\nu + g^{\mu \nu}) \partial_\nu \left( \frac{\mu}{T} \right) \]

Assume conformal symmetry

\[ \varepsilon = 3p = T^4 f(\mu/T) \quad n = \mu T^2 g(\mu/T) \]
Gubser flow

A boost-invariant, radially expanding exact solution

\[ \varepsilon = \frac{f(\alpha)C^4}{\tau^4 \cosh^8 \rho} \lesssim \frac{1}{\tau^{4/3}} \frac{1}{(L^4 + 2(\tau^2 + x_\perp^2)L^2 + (\tau^2 - x_\perp^2)^2)^{4/3}} \]

\[ n = \frac{\alpha g(\alpha)C^3}{\tau^3 \cosh^2 \rho} \]

\[ \rho = -\sinh^{-1} \frac{L^2 - \tau^2 + x_\perp^2}{2L\tau} \]

\[ \frac{\mu}{T} = \alpha = \text{const.} \]
Viscous Gubser flow at finite $\mu$

Solution valid to linear order in $\tilde{\eta} = \eta/s$

$$\varepsilon = T^4 f \left( \frac{\mu}{T} \right) \approx \frac{f(\alpha)C^4}{\tau^4 (\cosh \rho)^{8/3}} \left[ 1 + \frac{4h(\alpha)\tilde{\eta}}{9f(\alpha)C} \sinh^3 \rho \right. 2F1 \left( \frac{3}{2}, \frac{7}{6}, \frac{5}{2}; -\sinh^2 \rho \right) \right]$$

$$n = \mu T^2 g \left( \frac{\mu}{T} \right) = \frac{\alpha g(\alpha)C^3}{\tau^3 \cosh^2 \rho}$$

$$h(\alpha) = \frac{4}{3} f(\alpha) - \alpha^2 g(\alpha)$$

$$\frac{\mu}{T} = \alpha + \frac{2h\tilde{\eta}}{Cf} \left( \frac{L^2 + x_{\perp}^2}{2L\tau} \right)^{2/3} \left( \frac{4}{3\alpha} + \frac{4g'}{3g} - \frac{f'}{f} \right)$$
Anisotropic viscous solution at finite $\mu$

Gubser, Yarom (2011)

Add $\cos n\phi$ perturbations to the viscous Gubser flow.

$$\varepsilon \rightarrow \varepsilon(1 + \varepsilon_n A \cos n\phi),$$

$$u_\perp \rightarrow u_\perp + \varepsilon_n B \cos n\phi$$

Solve the linearized equations for $A, B, ...$
Perturbed solution at early time

At early times, $\tau < L$, the linearized equations are analytically tractable

$$
\varepsilon \approx \frac{C^4}{\tau^{4/3}} \frac{(2L)^{8/3}}{(L^2 + x_\perp^2)^{8/3}} \left( 1 - \frac{2\eta/s}{3C} \left( \frac{L^2 + x_\perp^2}{2L\tau} \right)^{2/3} \right)^4 \times \left[ 1 - 4\varepsilon_n \left( 1 + \frac{2\eta/s}{3C} \left( \frac{L^2 + x_\perp^2}{2L\tau} \right)^{2/3} \right) \left( \frac{2Lx_\perp}{L^2 + x_\perp^2} \right)^n \cos n\phi \right]
$$

$$
u_\perp = \frac{2\tau x_\perp}{L^2 + x_\perp^2} + \varepsilon_n \frac{3nL\tau}{L^2 + x_\perp^2} \left( \frac{2Lx_\perp}{L^2 + x_\perp^2} \right)^{n-1} \frac{L^2 - x_\perp^2}{L^2 + x_\perp^2} \cos n\phi,
$$

$$
u_\phi = -\varepsilon_n \frac{3n\tau}{2} \left( \frac{2Lx_\perp}{L^2 + x_\perp^2} \right)^n \sin n\phi
$$
Computing $\nu_n$ analytically

$$(2\pi)^3 \frac{dN^\pm}{dY p_T dp_T d\phi_p} = \int_\Sigma (-p^\mu d\sigma_\mu) \left( \exp \left( \frac{u \cdot p \pm \mu}{T} \right) + \delta f \right)$$

constant energy surface

Knudsen number $K \propto \eta/s$

Freezeout time $\tau_f(x_\perp, \phi) \approx \frac{(2L)^5}{B^3 (L^2 + x_\perp^2)^2} \left( 1 - \frac{3K(L^2 + x_\perp^2)^2}{2(2L)^4} \right) - 3\epsilon_n \left( \frac{2Lx_\perp}{L^2 + x_\perp^2} \right)^n \cos n\phi$

$\mu = \frac{2h\bar{\eta}}{Cf} \left( \frac{L^2 + x_\perp^2}{2L^2} \right)^{2/3} \left( \frac{4g}{3\alpha} + \frac{4g'}{3g'} - \frac{f'}{f} \right) \left( 1 + \epsilon_n \left( \frac{2Lx_\perp}{L^2 + x_\perp^2} \right)^n \cos n\phi \right)$

Work in the regime $B^3 \sim L/\tau \gg 1$

Analytically compute the CF integral to leading order in $1/B^3$
The result

YH, Noronha, Torrieri, Xiao (2014)

extension to finite density: YH, Monnai, Xiao (2015)

\[
\frac{v_n^\pm}{\varepsilon_n} = \frac{9}{64} \frac{\Gamma(3n)}{\Gamma(4n)} \left( \frac{128}{B^3} \right)^n \Gamma^2 \left( \frac{n}{2} \right) \frac{n^2(3n + 2)^2(n - 1)}{2(4n + 1)}
\]

\[
+ \frac{27K}{256} \frac{\Gamma(3n)}{\Gamma(4n)} \left( \frac{128}{B^3} \right)^n \Gamma^2 \left( \frac{n}{2} \right) \frac{n^3(n - 1)}{3n - 1} \left\{ -\frac{1}{4}(3n^2 + 3n + 2) + \frac{\mu}{2T} \left( \frac{3n}{2} + 1 \right) \left( \pm 1 - \frac{3f'}{4f} \right) \right\}
\]

\[
v_n^{X^+} - v_n^{X^-} \approx \frac{3n(4n + 1)K}{4(3n - 1)(3n + 2)T} (\mu_B B + \mu_S S + \mu_I I) v_n^{X,\text{ideal}}
\]
\[ \frac{v_n}{\epsilon_n} = \frac{9}{64 \Gamma(4n)} \left( \frac{128}{B^3} \right)^n \Gamma^2 \left( \frac{n}{2} \right) \frac{n^2(3n + 2)^2(n - 1)}{2(4n + 1)} + \cdots \]
A comparison with the STAR BES data

\[ v_2^{X^+} - v_2^{X^-} \approx \frac{27K}{80T} (\mu_B B + \mu_S S + \mu_I I) v_2^{X,\text{ideal}} \]

\[ \Delta v_2^p > \Delta v_2^\Lambda > \Delta v_2^\Xi > \Delta v_2^K > 0 > \Delta v_2^\pi \]

\[ \eta/s = 0.2 \]
\[ \mu_S/\mu_B = 0.23 \]
\[ \mu_I/\mu_B = -0.15 \]

Too large.
Need other mechanisms?
\[ \Delta v_2^\pi \propto A_{ch} \] from `normal’ hydro?

\[ \Delta v_2^\pi = v_2^{\pi^-} - v_2^{\pi^+} \approx rA_{ch} \quad \leftrightarrow \quad \Delta v_2^\pi \approx \frac{-\mu_I}{T} \frac{27}{80} K v_2^{ideal} \]

Looks similar, if we can relate \( A_{ch} \propto \mu_I \) in some sense.

In the STAR measurements, each bin of \( A_{ch} \) contains \( \mathcal{O}(10^5) \) events. Assign effective chemical potentials \( \mu_B, \mu_S, \mu_I \) in each bin.
$A_{ch}$ in the resonance gas model

Include all hadronic resonances below 2GeV.

$$A_{ch} = c(T)\mu_B + c'(T)\mu_I + c''(T)\mu_S$$

$\Delta v_2^\pi = v_2^{\pi^-} - v_2^{\pi^+} \approx rA_{ch} \quad \leftarrow \quad \Delta v_2^\pi \approx \frac{-\mu_I}{T} \frac{27}{80} K v_2^{ideal}$

$r > 0$ from the STAR data $\Rightarrow$ Sign mismatch!
Reversing the sign

$\mu_B, \mu_S, \mu_I$ not independent of each other in heavy-ion

Assume the empirical relations from thermal fits and lattice

$\mu_I \approx -0.03 \mu_B$

$\mu_S \approx 0.25 \mu_B$

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Bazavov et al. PRL109 (2012) 192302
Fitting the STAR data

Adjust the Knudsen number $K$ to fit the slope at 35% centrality

$\rightarrow$ Consistent with the previously extracted value in Bhalerao, Blaizot, Borghini, Ollitrault PLB627 (2005) 49

The centrality/energy dependence of the slope $r \sim \frac{\varepsilon_2}{S_\perp} \frac{dN}{dy}$
Kaons and $\nu_3$

$$\frac{\Delta v_2^K}{v_{K, ideal}} \approx -\frac{27K}{80} (\mu_S S + \mu_I I)$$

→ The slope for kaons is negative

$\nu_3$

$$v_3^{X^+} - v_3^{X^-} \approx \frac{117K}{352T} (\mu_B B + \mu_S S + \mu_I I) v_3^{X, ideal}$$

$$\frac{\Delta \nu_3}{\nu_3} = 0.985 \frac{\Delta \nu_2}{\nu_2}$$

$$r_3 \approx \frac{\nu_3}{\nu_2} r_2$$
slope positive also for kaons!  
very small slope for $v_3$

The $p_T$ range used in these analyses is rather small.

$0.15 < p_T < 0.5 \text{GeV/c}$
Discussions

• STAR pt range $0.15 < p_T < 0.5 \text{GeV/c}$ OK for pion $v_2$, but not for kaons and $v_3$. ($v_n$ dominated by $p_T \sim nT$.)

  STAR has updated their data $\rightarrow$ Qi-Ye Shou’s talk tomorrow

• Quantum statistics effect increases the magnitude of $r$. There may be many other effects.

• Positive $r$ for kaons very hard to understand in `normal’ hydro.