Non-equilibrium dynamics of topological transitions and axial charges

Soeren Schlichting

Based on Mark Mace, SS, Raju Venugopalan arXiv:1601.07342

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BROOKHAVEN NATIONAL LABORATORY
Motivation

• Quantitative understanding of anomaly induced transport phenomena (CME, CMW, CVE, …) in heavy-ion collisions is essential to guide experimental searches for these effects

• Chiral-magnetic effect

\[ j_v \propto n_5 B \]

\( n_5 \): axial charge density

\( B \): magnetic field


• While theoretical estimates of the magnetic field are becoming more and more reliable, significant uncertainty surrounds axial charge density
Axial charge & axial anomaly

- Imbalances of axial charge density ($n_5$) are sourced by fluctuations of the non-abelian field strength tensor due to the axial anomaly

$$\partial_\mu j_{5,f}^\mu = 2m_f \bar{q}\gamma_5 q - \frac{g^2}{16\pi^2} F_{\mu\nu}^a \tilde{F}_{\nu}^{\mu a}$$

  - axial current
  - quark mass
  - field-strength fluctuations
    $$\propto \vec{E} \cdot \vec{B}$$

- Considering the contribution to local imbalances of the axial charge $n_5(x,t)$ one can distinguish

  - contributions due to topological transitions (sphalerons)

  - ordinary fluctuations of $\vec{E} \cdot \vec{B}$ due to finite temperature/non-equilibrium excitations
Axial charge & axial anomaly

• Considering local quantities \( \partial_{\mu}j_{5}^{\mu} \) both topological transitions and ordinary field strength fluctuations contribute significantly.

• Equilibrium dynamics in the long wave-length and long time limit is dominated by sphaleron transitions.

• Sphaleron transitions characterized by integer change of the Chern-Simons number

\[
\Delta N_{CS} = \frac{g^2}{8\pi^2} \int d^4 x \, \vec{E}_a \vec{B}_a
\]

\( \rightarrow \) Induce global imbalances of axial charge

\[
J_{5}^{0} = \int d^3 x \, j_{5}^{0} \quad \Delta J_{5}^{0} = -2N_f \Delta N_{CS}
\]

Contributions to \( \vec{E} \cdot \vec{B} \)

Mace, SS, Venugopalan

arXiv:1601.07342
Sphaleron transitions in thermal equilibrium

- Sphaleron transition rate dominated by modes on the order of the magnetic screening length ($\sim 1/g^2 T$) 
  \[
  \Gamma_{sph}^{eq} = \kappa \alpha_S^5 T^4
  \]

- Individual sphaleron transition are uncorrelated with each other on sufficiently long time scales $\rightarrow$ Probabilistic picture

  Chern-Simons number ($\propto J^0_5$) exhibits an integer random walk

  Diffusion constant given by $\Gamma_{sph}^{eq}$

  \[
  (\alpha_S T)^3 \sim \text{phase space density}
  \]

  \[
  \alpha^2_S T \sim \text{typical frequency}
  \]
Challenges with regard to heavy-ion collisions

• Since life time of magnetic field is presumable very short (~0.1-1 fm/c) system is out-of-equilibrium during the time scales relevant for CME & Co.

  Determine the sphaleron transition rate in out-of-equilibrium plasma

• Since time scales involved are rather short it is not clear wether topological transitions actually provide the dominant contribution to CME & Co.

  Study the real-time dynamics of fermions in out-of-equilibrium plasma in the presence of a magnetic field
Early-stages of HIC

- Early time Glasma characterized by non-perturbatively large phase-space occupancy of gluons $f(p \sim Q_s) \sim 1/\alpha_s$ with typical momenta $\sim Q_s$

- Initially no separation between hard scale, electric and magnetic screening scale

\[
\begin{align*}
\text{Equilibrium} & : \Lambda \sim T & m_D \sim gT & \sqrt{\sigma} \sim g^2T \\
\text{Glasma} & : \Lambda \sim Q_s & m_D \sim Q_s & \sqrt{\sigma} \sim Q_s
\end{align*}
\]

- Naturally expect enhancement of sphaleron rate in the Glasma

- Will present classical Yang-Mills simulations of non-perturbative early time dynamics, but neglect longitudinal expansion for simplicity
Classical Yang-Mills & topology on the lattice

- Solve classical Yang-Mills equations on a space-time lattice to study non-equilibrium dynamics of Glasma

- Extract Chern-Simons number $N_{CS}$ by evaluating

$$\frac{dN_{CS}}{dt} = \frac{g^2}{8\pi^2} \int d^3x \ E_i^\alpha(x) B_i^\alpha(x)$$

- Since local operator definition of $\vec{E}_a \cdot \vec{B}_a$ is not a total derivative on the lattice, we use cooling to remove UV fluctuations and isolate topological transitions.
Sphaleron transitions in the Glasma

Evolution of the Chern-Simons number for a single configuration

Histograms of Chern-Simons number difference

Can identify sphaleron transitions and separate from fluctuations by varying the amount of cooling

Mace, SS, Venugopalan arXiv:1601.07342

Mace, SS, Venugopalan arXiv:1601.07342
Chern-Simons number Histograms

- Early times dominate generation of axial charge imbalance
Quantifying the sphaleron rate

Extract auto-correlation function of the Chern-Simons number

\[
\frac{1}{V} \langle (N_{CS}(t + \delta t) - N_{CS}(t))^2 \rangle = \Gamma_{sph}^e q \delta t
\]

Equilibrium:

\[
\Delta N_{cs\, auto-correlation} = \langle (N_{CS}(t + \delta t) - N_{CS}(t))^2 \rangle / (g^2 T^3 V)
\]

- Simple probabilistic picture not applicable in the Glasma — clear non-Markovian emerge from the auto-correlation function

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Glasma:

Mace, SS, Venugopalan arXiv:1601.07342
Quantifying the sphaleron rate

Define non-equilibrium sphaleron rate by the early rise of the auto-correlation function

\[ \Gamma_{sph}^{neq}(t) = \left( \frac{(N_{CS}(t+\delta t) - N_{CS}(t))^2}{V \delta t} \right)_{Q_s \delta t < 10} \]

Non-equilibrium sphaleron rate

- Sizeable contribution from field strength fluctuations
- Strong time dependence observed — rate is largest at early times and decreases rapidly as a function of time

Mace, SS, Venugopalan arXiv:1601.07342
Do we understand the non-equilibrium sphaleron rate?

Since in equilibrium $\Gamma_{sph}$ is controlled by modes on the order of the magnetic screening scale $\sim \sqrt{\sigma}$, one should really compare $\frac{\Gamma_{sph}}{\sigma^2}$

**Characteristic scales**

- Equilibrium
  - $\sim T^2$
  - $\sim g^2 T^2$
  - $\sim g^4 T^2$

- Separation of scales emerges dynamically

**Sphaleron rate**

$\Gamma_{sph}/\sigma^2$

- Non-equilibrium sphaleron rate dominated by modes on the order of the magnetic screening scale
Sphaleron rate in the Glasma

• Within a simplified description of the Glasma (neglecting the longitudinal expansion) we find that the rate of topological transitions is significantly enhanced at early times

\[ \Gamma_{sph}^{neq} \sim Q_s^4 \quad \text{Glasma:} \quad \Gamma_{sph}^{eq} \sim \alpha_s^5 T^4 \quad \text{Equilibrium:} \]

• Similar to the thermal case the non-equilibrium sphaleron rate is dominated by soft modes on the order of the magnetic screening scale

\[ \Gamma_{sph}^{neq}(t) \approx 2 \times 10^{-3} \sigma^2(t) \quad \text{where} \quad \sigma^2(t) \sim Q_s^2(Q_st)^{-2/3} \]

for the non-expanding case.

\[ \rightarrow \text{Early times dominate generation of axial charge imbalance} \]

• On the time scales of interest non-equilibrium behavior of N_{CS} show qualitative differences to the thermal case

I) Significant contribution from fluctuating field strength
II) Non-Markovian nature of sphaleron transitions
Implications for phenomenology

• Generalization of results to the longitudinally expanding case

\[ \Gamma_{sph}^{neq} \sim Q_s^4 \quad \text{Glasma:} \quad \Gamma_{sph}^{eq} \sim \alpha_S^5 T^4 \quad \text{Equilibrium:} \]

Still expect sphaleron transition rate to be largely enhanced at early times and decrease rapidly as a function of time

-> Early times (should) dominate generation of axial charge imbalance

• Quantitative statements are difficult to infer as several issues prevent straightforward extension to the longitudinally expanding case

I) Evolution of magnetic scale not know at present
II) Effect of momentum space anisotropy of the plasma
Outlook

• So far we have focused on gauge-field dynamics and studied only the right-hand side of the anomaly equation

\[ \partial_\mu j_{5,f}^\mu = 2m_f \bar{q} \gamma_5 q - \frac{g^2}{16\pi^2} F^\alpha_{\mu\nu} \tilde{F}^{\mu\nu}_\alpha \]

• Except for computational cost no fundamental limitation in studying dynamics of fermions in real-time lattice simulations

(work in progress with Niklas Mueller, Sayantan Sharma, Mark Mace)

Axial density during sphaleron transition

Axial anomaly at the center

\[ (i \gamma^\mu D_\mu - m) \hat{\Psi} = 0 \]

\[ \hat{\Psi} = \Psi - \sum_i \lambda_i \phi_1 \phi_2 \hat{\phi}_i \]

\[ \Delta_{\text{CS}} \rightarrow \text{progress towards real-time lattice simulations of CME} \]
Backup
String tension

Wilson loops

Derivative w.r.t area
Topology measurement
Topology measurement

![Diagram of real-time evolution and calibration path sum over time](image)

- At $t=0$, the initial state is shown.
- As $t$ increases, the state evolves through cooling and calibration processes.
- At $t=T_c$, the system reaches a critical point.
- At $t=\infty$, the state is in the vacuum.

![Graph showing calibration path sum over time](image)

- The graph plots the calibration path sum against time $Q_s t$.
- The data points suggest a linear trend with a slope of $Q_s^2 \tau_c = 1$ and $Q_s \tau_c = 12$. 

The figure illustrates the dynamic evolution of a system through various stages, highlighting the importance of calibration and the measurement of topology in understanding the system's behavior.
Dependence on IC and Cooling depth
Volume independence