EXCITATION OF CHANDRASEKHAR–KENDAL STATES IN QUARK GLUON PLASMA

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1. Electromagnetic fields in relativistic heavy-ion collisions:
   (i) Initial conditions
   (ii) Effect of anomaly
   (iii) Quantum vs classical sources

2. Photons with definite angular momentum and magnetic helicity as tool to study the chiral anomaly
PART I: CLASSICAL EM FIELD IN QGP
1. At $t < t_0$ EM field in vacuum is produced only by valence charges (external sources).
2. At $t > t_0$ there are two contributions:
   (i) Initial field (at $t = t_0$) that evolves in QGP, $B_{\text{init}}$
   (ii) Field due to the external sources, $B_{\text{val}}$

Previous work: $t_0 = 0$ meaning that $B_{\text{init}}$ is neglected.

In fact, $B_{\text{init}}$ is often a dominant component of the field.
\[ \nabla^2 \mathbf{A}(\mathbf{r}, t) = \partial_t^2 \mathbf{A}(\mathbf{r}, t) + \sigma \partial_t \mathbf{A}(\mathbf{r}, t) - \mathbf{j}(\mathbf{r}, t) \]

(external current)

(assumed to be constant)

\[ \mathbf{A}(\mathbf{r}, t_0) = \hat{z} \Phi(\mathbf{r}, t_0) \]

Vector potential and its time-derivative in vacuum at \( t=t_0 \)

\[ \partial_t \mathbf{A}(\mathbf{r}, t_0) = \hat{z} \Psi(\mathbf{r}, t_0) \]

In momentum space (for a single charge)

\[ -k^2 \mathbf{A}_k(t) = \partial_t^2 \mathbf{A}_k(t) + \sigma \partial_t \mathbf{A}_k(t) - ev\hat{z}e^{-ik_zvt}, \]

\[ \mathbf{A}_k(t_0) = \hat{z} \Phi_k(t_0) = \frac{ev\hat{z}}{k_z^2/\gamma^2 + k_\perp^2} e^{-ik_zvt_0}, \]

\[ \partial_t \mathbf{A}_k(t_0) = \hat{z} \Psi_k(t_0) = -ik_zv \frac{ev\hat{z}}{k_z^2/\gamma^2 + k_\perp^2} e^{-ik_zvt_0}. \]
DIFFUSION APPROXIMATION

For an ultrarelativistic charge

\[ \partial_t^2 - \partial_z^2 \sim \frac{k_z^2}{\gamma^2} \ll k_{\perp}^2 \sim \sigma k_z \]

Implying that \( \sigma \gamma \gg k_{\perp} \) a good approximation for not too small impact parameters \( b \sim 1/k_t \)

Introduce a new “time-variable” \( \lambda(t) = \int_{t_0}^{t} \frac{dt'}{\sigma(t')} \)

Equation of motion \(-k_{\perp}^2 A_k = \partial_\lambda A_k - j_k\) with the appropriate boundary condition is solved by

\[
A_k(t) = \hat{z} \left\{ \frac{ev}{\sigma} \frac{1}{k_{\perp}^2 - i k_z v} \left( e^{-ik_z vt} - e^{-\frac{k_{\perp}^2}{\sigma}(t-t_0)} e^{-ik_z vt_0} \right) + \Phi_k e^{-\frac{k_{\perp}^2}{\sigma}(t-t_0)} \right\}
\]

\[ \text{valence current} \]

\[ \text{initial field (that at } t=t_0 \text{ permeated the plasma)} \]
MAGNETIC FIELD IN CONFIGURATION SPACE

\[ B = B_{\text{val}} + B_{\text{init}} \]

\[ eB_{\text{val}}(r, t) = \hat{\phi} \frac{\alpha \pi b}{2\sigma(z/v)[\lambda(t) - \lambda(z/v)]^2} \exp \left\{ -\frac{b^2}{4[\lambda(t) - \lambda(z/v)]} \right\} \theta(tv - z)\theta(z - vt_0), \]

\[ eB_{\text{init}}(r, t) = \hat{\phi} \gamma \alpha v \int_0^\infty dk_\perp J_1(k_\perp b) \exp \left\{ -k_\perp^2 \lambda(t) - k_\perp \gamma |z - vt_0| \right\} \]

1. Note, that \( B_{\text{val}}(r,t_0)=0 \), while \( B_{\text{init}}(r,t_0) \) reproduces the initial field.

2. Energy dependence: \( B_{\text{init}}(r,t) \sim \gamma \), while \( B_{\text{val}}(r,t) \sim \gamma^0 \)

3. Late time-dependence: \( B_{\text{init}}(r,t) \sim 1/t^{3/2} \), while \( B_{\text{val}}(r,t) \sim 1/t^2 \)
FIG. 2: Magnetic field in units of $m_\pi^2/e$. $\sigma = 5.8$ MeV, $z = 0$ fm ($\eta = 0$). Left panel: $t_0 = 0.2$ fm; right panel $t_0 = 0.5$ fm. Valence current does not contribute at all ($B_{val} = 0$).
Namely, expansion is supposed to be isentropic. To see the impact of Fig. 4.

Since magnetic field in vacuum decreases as $1/t$, becomes smaller than independent of the initial value of magnetic field at $z = 0$. Because of the step functions means that the magnitude of the field in the former case is about 15 times larger than in the later.

Energy dependence of magnetic field between the RHIC and LHC energies can be seen in Fig. 3 is similar to Fig. 2 except that $t_0 = 0.2$ fm, right panel: $t_0 = 0.5$ fm. Solid, dashed and dotted lines stand for $B$, $B_{\text{init}}$ and $B_{\text{val}}$.

FIG. 4: Magnetic field in units of $m_{\pi}^2/e$. $\sigma = 5.8$ MeV, $z = 0.2$ fm. Solid, dashed and dotted lines stand for $B$, $B_{\text{init}}$ and $B_{\text{val}}$. Left panel: $\gamma = 100$ (RHIC), right panel: $\gamma = 2000$ (LHC).
Two models:

\[ \sigma(t) = \frac{\sigma}{2^{-1/3}(1 + t/t_0)^{1/3}} , \quad \text{Model A.} \]

\[ \sigma(t) = \sigma \left(1 - e^{-t/\tau}\right) , \quad \text{Model B} \quad \tau=1\text{fm} \]

FIG. 5: Magnetic field in units of $m_\pi^2/e$. $z = 0.2$ fm $t_0 = 0.2$ fm. Left panel: model A. Right panel: model B. Solid, dashed and dotted lines stand for $B$, $B_{\text{init}}$ and $B_{\text{val}}$.

Time-dependence of conductivity is important at later times.
FIG. 2: Magnetic field of a point charge as a function of time $t$ at $z = 0$. (Free space contribution is not shown). Electrical conductivity $\sigma = 5.8$ MeV. Solid line on both panels corresponds to $B = B_\phi$ at $\sigma_\chi = 0$. Broken lines correspond to $B_\phi$ (dashed), $B_r$ (dashed-dotted) and $B_z$ (dotted) with $\sigma_\chi = 15$ MeV on the left panel and $\sigma_\chi = 1.5$ MeV on the right panel. Note that the vertical scale on the two panels is different.
QUANTUM SOURCES WITH CLASSICAL SOURCES

- Classical sources: point-like particles with definite momentum and coordinates
- State with definite momentum (plane waves) are also not appropriate because of the large interaction range (em force!)
- Need a realistic wave function; we took a 1 fm wide Gaussian.
- Spin neglected (scalar QED)
1. The fact that QGP emerges when the space is already permitted by the EM field has a significant impact on the EM field strength and spacetime structure.

2. There are two contributions to the classical magnetic field in plasma:
   (i) The initial field that evolves in QGP, $B_{\text{init}}$
   (ii) The field due to the external sources, $B_{\text{val}}$

   Which contribution dominates depends on the kinematics.

3. Chiral conductivity does not have a dramatic effect on $B$ in QGP, see however later in this talk.

4. Point charge approximation of the sources is not accurate.
PART II: EMISSION OF PHOTONS IN CK STATES
Chiral anomaly: \( \mathbf{j} = \sigma_\chi \mathbf{B} \) in QGP and zero otherwise.

If QGP is sphere with a sharp boundary:

\[
\nabla \cdot \mathbf{B} = 0, \\
\nabla \times \mathbf{B} = \sigma_\chi \mathbf{B} \quad \Rightarrow \quad \text{Chernodub’s knots} \\
\hat{r} \cdot \mathbf{B} \big|_{\partial D} = 0
\]

This model is naive. However, it points out that the CK states are a natural basis to discuss the topological properties of EM field in QGP.

Indeed, Hirono, Kharzeev and Yin used the CK states to follow the evolution of the magnetic helicity in time.

What is the spectrum of the CK states produced in QGP?
SPHERICAL EM WAVES WITH MAGNETIC HELICITY

\[ \nabla^2 A - \partial^2_t A = 0 \]
\[ A_{klm}^h(r, t) = h k W_{klm}^h(r) e^{-i \omega_k t} \]

where
\[ \nabla \times W_{klm}^h(r) = h k W_{klm}^h(r) \]

\[ W_{klm}^h(r) = \frac{1}{\pi \sqrt{2k}} \left( T_{klm}^h(r) - i h P_{klm}^h(r) \right) \]

\[ T_{klm}^h(r) = \frac{j_l(kr)}{\sqrt{l(l+1)}} L[Y_{lm}(\theta, \phi)], \quad P_{klm}^h(r) = \frac{i}{k} \nabla \times T_{klm}^h(r) \]

Plane wave expansion:
\[ W_{klm}^h(r) = \sum_\lambda \int \frac{d^3q}{(2\pi)^3} e^{i q \cdot r} \epsilon_{q\lambda} w_{klm}^h(q, \lambda) \]
\[ w_{klm}^h(q, \lambda) = \frac{1}{\pi \sqrt{2k}} \epsilon^\ast_{q\lambda} \cdot \left( T_{klm}^h(q) - i h P_{klm}^h(q) \right) \]
\[ T_{klm}^h(q) = \frac{2\pi^2}{k^2} (-i)^l \frac{1}{\sqrt{l(l+1)}} \delta(k - q) L Y_{lm}(\hat{q}) \]
\[ P_{klm}^h(q) = -\frac{1}{k} q \times T_{klm}^h(q) \]
QUANTIZATION OF THE EM FIELD

\[
A(\mathbf{r}, t) = \sum_{klmh} \left( \hbar k a^h_{klm} W^h_{klm}(\mathbf{r}) e^{-i\omega_k t} + c.c. \right)
\]

\[
E(\mathbf{r}, t) = -\partial_t A(\mathbf{r}, t) = \sum_{klmh} \left( i\hbar \omega_k a^h_{klm} W^h_{klm}(\mathbf{r}) e^{-i\omega_k t} + c.c. \right),
\]

\[
B(\mathbf{r}, t) = \nabla \times A(\mathbf{r}, t) = \sum_{klmh} \left( k^2 a^h_{klm} W^h_{klm}(\mathbf{r}) e^{-i\omega_k t} + c.c. \right)
\]

Total energy: 
\[
\mathcal{E} = \frac{1}{2} \int (E^2 + B^2) d^3 r = \sum_{klm} \omega_k a^h_{klm} a^{h*}_{klm}
\]

Total magnetic helicity: 
\[
H = \int A \cdot B \, d^3 r = \sum_{klm} h a^h_{klm} a^{h*}_{klm}
\]

Promote \(a^h_{klm}\) to operators that create/annihilate the CK photon states.
ROLE OF ELECTRIC CURRENTS

Throughout the discussion of the CK photons I neglected the Ohm’s and anomalous currents for three reasons.

\[ \nabla \cdot B = 0, \quad \Rightarrow \quad -\nabla^2 A = -\partial_t^2 A - \sigma \partial_t A + \sigma \chi \nabla \times A \]

\[ \nabla \cdot E = 0, \]

\[ \nabla \times E = -\partial_t B, \]

\[ \nabla \times B = \partial_t E + \sigma E + \sigma \chi B \]

1. Phenomenology: \( k > 1/5 \text{fm} \). \( \sigma = 5-6 \text{ MeV} \Rightarrow k \gg \sigma \). Thus, for QGP the free-field approximation is good.

2. At finite \( \sigma \) EM energy is not conserved, at finite \( \sigma \chi \) magnetic helicity is not conserved. These problems are not too bad for hard scattering processes.

3. Problem: models with finite space-uniform \( \sigma \chi \) are acausal.
PROBLEM WITH CAUSALITY

From my last year presentation: magnetic field of a point charge.

\[
B = \int \frac{d^2 k \perp}{(2\pi)^2} e^{i k \perp \cdot b} \left\{ \frac{i}{\omega_2 - \omega_1} \left[ e^{-i \omega_1 x} - f(\omega_1) \theta(k \perp - \sigma_x) - e^{-i \omega_2 x} f(\omega_2) \right] \theta(x_-) \\
- \frac{i}{\omega_2 - \omega_1} e^{-i \omega_1 x} f(\omega_1) \theta(\sigma_x - k \perp) \theta(-x_-) \right\}
\]

acausal term is a manifestation of instability or model artifact?

One can write the medium response to the electric field as

\[
D(r, t) = E(r, t) + \int_0^\infty d\tau \int d^3 r' G(\tau, r') E(r - r', t - \tau)
\]

\[
D_{k\omega} = \epsilon(k, \omega) E_{k\omega} \quad \text{if there is no spatial dispersion (no k-dependence), then}
\]

\[
G(t, r) = \delta(r) \int_{-\infty}^\infty \frac{d\omega}{2\pi} \left[ \epsilon(\omega) - 1 \right] e^{-i \omega t} = \delta(r) G(t) \quad \text{causal response}
\]

\[
\epsilon(\omega) - 1 = \frac{i \sigma}{\omega}
\]
However, a relationship between $\mathbf{H}$ and $\mathbf{B}$ fields is such that

$$G_m(t, \mathbf{r}) = \pm \delta(t) \int_{-\infty}^{\infty} \frac{d^3 k}{(2\pi)^3} \frac{\sigma_{\chi}}{k} e^{i k \cdot \mathbf{r}} = \delta(t) G_m(\mathbf{r}) \quad \text{acausal response}$$

In fact, it can be shown that

$$\mathbf{H}(\mathbf{r}, t) = \mathbf{B}(\mathbf{r}, t) \pm \sigma_{\chi} \int \frac{d^3 r'}{|\mathbf{r} - \mathbf{r}'|^2} \mathbf{B}(\mathbf{r}', t)$$

Not good. Need a better model with spatial dispersion.
To excite the CK states in a controllable way consider scattering of a high energy quark off a thermal gluon

\[ q(p'^\mu) + g(k'^\mu) \rightarrow q(p'^\mu) + \gamma_{\text{CK}}(k'lmh) \quad \varepsilon \gg T \]

Scattering matrix

\[ S_{fi} = e_q g \int d^4 x \int d^4 y \bar{\psi}_f(x) \left[ iA^{h*}_{k'lm}(x) iS(x - y) (-iA^a_{k\lambda}(y)) \right. \\
\left. + (-iA^a_{k\lambda}(x)) iS(x - y) (iA^{h*}_{k'lm}(y)) \right] \psi_i(y), \]

Gluon field

\[ A^a_{k\lambda}(r) = \frac{1}{\sqrt{2\omega V}} t^a \epsilon_{k\lambda} e^{-ik\cdot x^\mu}, \]

\[ S_{fi} = \frac{-i(2\pi)h k'^\mu t^a}{\sqrt{2\varepsilon'} \sqrt{2\varepsilon} \sqrt{2\omega V}^{3/2}} \sum_{\chi'} \int d^3 q' w^{h*}_{k'lm}(q', \chi') M_C(qg \rightarrow q\gamma) \delta^4(p'^\mu + k'^\mu - p'^\mu - q'^\mu) \]

Plane wave decomposition of CK states

Compton scattering amplitude
where the Compton scattering amplitude is given by:

\[
|S_{fi}|^2 = \frac{(2\pi)^2 k^2 t^a t^a}{2 \varepsilon' 2 \varepsilon 2 \omega V^3} \delta(\varepsilon + \omega - \varepsilon' - \omega'_k) \frac{t}{2\pi} \left| \sum_{\lambda'} w_{k'lm}^{h*} (p + k' - p', \lambda') \mathcal{M}_C (qg \rightarrow q\gamma) \right|^2
\]

Rate

\[
\pi \frac{dR_{lm}^h}{dk'} = \frac{1}{4} \frac{1}{2N_c} \frac{(2\pi) k'^2}{2 \varepsilon' 2 \varepsilon 2 \omega V} \sum_{\lambda''} \left| \sum_{\lambda'} w_{k'lm}^{h*} (q', \lambda') \mathcal{M}_C (qg \rightarrow q\gamma) \right|^2 \frac{d^3 q'}{(2\pi)^3} f(\omega) \frac{d\Omega_k \omega^2}{(2\pi)^3}
\]

Gluon distribution

\[
f(\omega) = \frac{2(N_c^2 - 1)}{e^\omega/T - 1}
\]

In the UR limit quark masses can be neglected and the Compton amplitude splits into two chiral parts: one for the left-polarized and another for the right-polarized gluons; their contributions are equal:

\[
\left| \sum_{\lambda'} w_{k'lm}^{h*} (q', \lambda') \mathcal{M}_C \right|^2 \approx |\tilde{N}_{k'}|^2 |\mathcal{M}_C|^2 \sum_{\lambda'} \epsilon_{q'\lambda'} \cdot \left( T_{k'lm}^{h*} (q') + i\hbar P_{k'lm}^{h*} (q') \right)
\]
\[(p'^\mu)^2 = (p^\mu + k'^\mu - q'^\mu)^2 \Rightarrow p \cdot q' = p \cdot k \quad \text{for highly energetic quark } \varepsilon \gg \omega\]

The angles are related by \( \omega'_q(1 - \cos \chi) = \omega(1 - \cos \theta) \)

Dividing by flux density one gets the cross section

\[
\pi \frac{d\sigma_{lm}^h}{dk'} = \frac{e_q^2 g^2}{8(2\pi)^5 N_c \varepsilon' \varepsilon(1 - \cos \theta)} |N_{k'}|^2 f(\omega) \left| \sum_{\lambda'} \epsilon_{q'\lambda'} \cdot (T_{klm}^h(q') + i\hbar P_{klm}^h(q')) \right|^2 d^3q'd\Omega_k
\]

Integral over the angle \( \theta \):

\[
\int \frac{\omega d\Omega_k}{(e^{\omega/T} - 1)(1 - \cos \theta)} = 2\pi a \int_{1/2}^{\infty} \frac{dy}{e^{ay/T} - 1} = -2\pi T \ln(1 - e^{-a/2T})
\]
Polarization of the intermediate photon \[ \epsilon_{q'\lambda'} = \frac{1}{\sqrt{2}}(\hat{\xi} + i\lambda'\hat{\eta}), \]

\[ \hat{\xi} = \sin \phi \hat{x} - \cos \phi \hat{y}, \]
\[ \hat{\eta} = \cos \chi \cos \phi \hat{x} + \cos \chi \sin \phi \hat{y} - \sin \chi \hat{z}, \]
\[ \hat{\zeta} = \sin \chi \cos \phi \hat{x} + \sin \chi \sin \phi \hat{y} + \cos \chi \hat{z}. \]

Summation over the polarizations:

\[
\left| \sum_{\lambda'} \epsilon_{q'\lambda'} \cdot \left( T_{k'lm}^{h*}(q') + i\hbar P_{k'lm}^{h*}(q') \right) \right|^2 = 2 \left| \hat{\xi} \cdot \left( T_{k'lm}^{h*}(q') + i\hbar P_{k'lm}^{h*}(q') \right) \right|^2
\]
\[
= \frac{8\pi^4}{k'4l(l+1)} \delta(k' - q')^2 \left| (L\xi + i\hbar L\eta) Y_{lm}^{*}(q') \right|^2,
\]

In the xyz-plane

\[
-i(L\xi + i\hbar L\eta) = \frac{1}{2} e^{-i\phi}(1 + h \cos \chi)L_+ + \frac{1}{2} e^{i\phi}(-1 + h \cos \chi)L_- - h \sin \chi L_z
\]
Using the properties of the angular momentum operator

\[ L_{\pm} Y_{lm} = (L_x \pm iL_y)Y_{lm} = \sqrt{l(l+1) - m(m \pm 1)} Y_{l,m\pm 1} \]

write

\[ F_{lm}^h(x) = -\int \ln(1 - e^{-x(1-\cos \chi)/2}) \frac{1}{l(l+1)} |(L_\xi - i\hbar L_\eta) Y_{lm}(\hat{q}')|^2 d\Omega_{\hat{q}'} . \]

contains terms proportional to \( e^{i(m+2)\phi} , e^{-i(m+2)\phi} , e^{i\phi} \)

Using the orthogonality of the spherical harmonic on a unit circle in \( \phi \)

\[ F_{lm}^h(x) = -\frac{2\pi}{l(l+1)} \int_0^{\pi} d\chi \sin \chi \ln(1 - e^{-x(1-\cos \chi)/2}) \]

\[ \times \left\{ \sin^2 \chi m^2 |Y_{lm}(\chi, 0)|^2 + \frac{1}{4} (1 - h \cos \chi)^2 \left[ l(l+1) - m(m-1) \right] |Y_{l,m-1}(\chi, 0)|^2 \right. \]

\[ \left. + \frac{1}{4} (1 + h \cos \chi)^2 \left[ l(l+1) - m(m+1) \right] |Y_{l,m+1}(\chi, 0)|^2 \right\} . \]
**EXACT P–WAVE F–FUNCTIONS**

\( x = k/T \)

\[
F_{10}^{\pm 1}(x) = \frac{6}{x^5} \left[ 48 \text{Li}_6(e^{-x}) + 24x \text{Li}_5(e^{-x}) + 6x^2 \text{Li}_4(e^{-x}) + x^3 \text{Li}_3(e^{-x}) + x^3 \zeta(3) + 24x \zeta(5) \right]
- \frac{2\pi^4}{105x^5}(16\pi^2 + 21x^2),
\]

\[
F_{1,\pm 1}(x) = -\frac{3}{x^5} \left[ 192 \text{Li}_6(e^{-x}) + 120x \text{Li}_5(e^{-x}) + 34x^2 \text{Li}_4(e^{-x}) + 6x^3 \text{Li}_3(e^{-x}) + x^4 \text{Li}_2(e^{-x}) 
+ 72x \zeta(5) \right] + \frac{\pi^4(64\pi^2 + 35x^2)}{105x^5},
\]

\[
F_{1,\pm 1}^{\mp 1}(x) = -\frac{6}{x^5} \left[ 96 \text{Li}_6(e^{-x}) + 36x \text{Li}_5(e^{-x}) + 5x^2 \text{Li}_4(e^{-x}) + 3x^3 \zeta(3) + 60x \zeta(5) \right] 
+ \frac{\pi^2(128\pi^4 + 238\pi^2 x^2 + 105x^4)}{210x^5},
\]

**low x**

\( F_{10}^{\pm 1}(x) \approx \frac{3}{5} \ln \frac{1}{x}, \quad F_{1,\pm 1}^{\pm 1}(x) \approx F_{1,\pm 1}^{\mp 1}(x) \approx \frac{4}{5} \ln \frac{1}{x} \)

**high x**

\( F_{10}^{\pm 1}(x) \approx \frac{2\zeta(3)}{7x^2}, \quad F_{1,\pm 1}^{\pm 1}(x) \approx \frac{\pi^4}{3x^3}, \quad F_{1,\pm 1}^{\mp 1}(x) \approx \frac{\pi^2}{2x} \)
\[ F^h_{lm}(x) = -\frac{2\pi}{l(l+1)} \int_0^\pi d\chi \sin \chi \ln(1 - e^{-x(1-\cos \chi)/2}) \]

\[ \times \left\{ \sin^2 \chi m^2 |Y_{lm}(\chi, 0)|^2 + \frac{1}{4}(1 - h \cos \chi)^2 [l(l+1) - m(m-1)] |Y_{l,m-1}(\chi, 0)|^2 
+ \frac{1}{4}(1 + h \cos \chi)^2 [l(l+1) - m(m+1)] |Y_{l,m+1}(\chi, 0)|^2 \right\}. \]

At high x (k >> T) the main contribution comes from small angles

Expand

\[ Y_{lm}(\chi, \phi) = \sqrt{\frac{2l+1}{4\pi}} \left[ \delta_{m0} \mp \frac{e^{\pm i\phi}}{2} \sqrt{l(l+1)}\delta_{m,\pm 1}\chi - \frac{1}{4} l(l+1)\delta_{m0}\chi^2 
\mp \frac{e^{\pm 2i\phi}}{8} \sqrt{(l+2)(l+1)l(l-1)}\delta_{m,\pm 2}\chi^2 \right]. \]

\[ F^h_{lm}(x) \approx -\frac{2\pi}{l(l+1)} \int_0^\infty d\chi \chi \ln \left( 1 - e^{-x\chi^2/4} \right) [l(l+1) - m(m+h)] \frac{2l+1}{4\pi} \delta_{m,-h} \]

\[ = \frac{\pi^2}{6} \frac{1}{x} (2l+1) \delta_{m,-h}, \quad x \gg 1. \]

The leading term has m+h=0

In general \( F \sim 1/x^{|m+h|+1} \)
The cross section
\[ \frac{1}{R} \frac{d\sigma_{lm}^h}{dk'} = \frac{e_q^2 g^2 C_F}{16\pi^4} \frac{T}{k'\varepsilon^2} F_{lm}(k'/T), \]

Initial condition of Hirono, Kharzeev and Yin:

\[ B_{\text{Hopf}}(x, t) = \sqrt{\frac{4h_m l}{3\pi}} \int_0^\infty dk k^2 e^{-kL_{\text{EM}}} \left[ (kL_{\text{EM}}^2)W_{11}^+(x, t)e^{-ikt} + \text{c.c.} \right] \]
If $N_L - N_R \neq 0$, then there is an asymmetry between $h=1$ and $h=-1$

$$\sigma^h_{lm}(R) = \sigma^{+1}_{lm}(\text{tot})$$

$$\sigma^h_{lm}(L) = \sigma^{-1}_{lm}(\text{tot})$$

This implies excess of either $m=-1$ or $m=1$ photons.
1. CK photons (i.e. photons in states with definite angular momentum and magnetic helicity) is a useful tool to study the anomaly-driven processes in which magnetic helicity changes in time.

2. As the first step, we computed the spectrum of these photons emitted by an ultrarelativistic quark in a non-anomalous plasma.

3. We are studying the interaction of the CK photons with anomaly and with the external classical field.
Quantum numbers of the CK photons scattering off the classical external field change (similarly to the Kerr effect for the plane waves).

Coupling of the CK photons to the external field comes through the higher order terms:

$$\beta_1 (E \cdot B)^2 \quad \beta_2 (E^2 - B^2)^2$$

where

$$\beta_{1,2} \propto \frac{\alpha^2}{M^4}$$

Naturally, $M \gtrsim 1/R$

In a strong external field these terms give large contributions $\sim (\beta_{1,2} B_{\text{cl}}^2)B^2$

The anomalous currents also couple to the external field, though the contribution is $\alpha$ times smaller.

*Work in progress ....*