Metastable vacuum states in QCD and P odd effects in heavy ion collisions (arxiv 1407.5121).

Ariel Zhitnitsky

University of British Columbia, Vancouver, Canada

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1. Motivation.

The main goal of this talk is to argue that local $P$, $CP$ violating fluctuations in QCD as observed at RHIC and the LHC is a consequence of

1. the quantum anomalies and
2. a long range order.

The presence of a long range order is a required feature for any explanations formulated in terms of the hydrodynamical parameters (anomalous transport coefficients).

Topic of the present talk focuses on a single question: What could be the nature and origin of this 4d long range correlation in gapped QCD with apparently a single scale $\Lambda_{QCD}$?
Let me reiterate this problem in the plain language: when one discusses hydrodynamics, kinetic coefficients, chiral fluid, transport coefficients, chiral magnetic potential, etc. one should always treat all these coefficients as constants (or slow variable parameters).

It implies that there must be a separation of scales between slow $\mu_5(\vec{x}, t)$ and typical QCD fast fluctuations. Otherwise, all these transport coefficients do not make sense (not defined).

“Long range order” in this talk implies the presence of a parametrically different scale separated from the QCD fluctuations.
Basic (sufficiently old) idea is: a large 4D domain with effective $\theta_{\text{ind}}(\vec{x}, t) \neq 0$ may be induced in heavy ion collisions (D. Kharzeev et al, 98, A. Zhitnitsky et al, 98).

$$\theta \frac{\alpha_s}{8\pi} G^{\alpha}_{\mu\nu} \tilde{G}^{\mu\nu\alpha} = \theta \partial_{\mu} K^{\mu}$$ is total derivative, does not change the equation of motion. Still, it leads to the physically observable effects.

There existence of these coherent P-odd configurations in 4D space with $\theta_{\text{ind}}(\vec{x}, t) \neq 0$ had been assumed (without much elaboration).

Effective description of these coherent 4D domains is formulated in terms of $\mu_5 = \dot{\theta}_{\text{ind}}(\vec{x}, t)$. Precisely this parameter $\mu_5$ enters all hydrodynamical anomalous transport coefficients. We want to understand the nature of coherence of $\mu_5$. 
If $\mu_5$ is indeed generated coherently on scales $L \gg \Lambda_{QCD}$ then:

For the uniform magnetic field the electric field will be induced along $B$ in the presence of large domain $\theta$ (assuming a large coherent effect)

$$L^2 E_{z}^{\text{ind}} = -\left(\frac{e \theta}{2\pi}\right) l, \quad \text{where} \quad l = \frac{e}{2\pi} \int d^2 x_\perp B_z^{\text{ext}}$$

$$[Q(z = +L) - Q(z = -L)] \sim \left(\frac{e \theta}{2\pi}\right) l$$

The induced electric field will lead to the induced currents and to the separation of charges along $B$ (CME) on macroscopic scale where $\mu_5$ is coherent

$$\vec{J} = (\mu_L - \mu_R) \frac{e \vec{B}}{2\pi^2}, \quad \text{where} \quad (\mu_L - \mu_R) = \dot{\theta}$$

CSE / CME / CVE / CESE / CMW / CVW... D. Kharzeev and A. Zhitnitsky, 2007; D. Kharzeev, L. McLerran, and H. Warringa, 2007; K. Fukushima, D. Kharzeev and H. Warringa, 2008... + many more...
2. Few known examples where $\mu_5$ is indeed generated on large scales

A number of coherent effects due to anomalies and large-sized topological solitons \((\text{Son & AZ 2004, Metlitski & AZ, 2005})\): axial current on a superfluid vortex, magnetization of the axial domain wall, EM currents on axial vortices, etc.

Large coherent $\mu_5$ could be also generated as a result of neutrino emission in the core of neutron stars \((\text{Charbonneau & AZ 2007, 2009})\), or core collapse supernova \((\text{Naoki Yamamoto, 2015})\)

There are many CM systems where $\mu_5$ might be coherently generated on large scales. For example, CME might have been observed \((\text{Kharzeev et al, 2014})\)
3. Strategy: Deformed QCD as a toy model

How could the long range structure ever occur in the gapped QCD system? We attempt to answer this question by using “deformed QCD” as a toy model.

This is a simplified version of QCD which, on one hand, is a weakly coupled gauge theory where computations can be performed in theoretically controllable manner.

On other hand, the corresponding deformation preserves all the relevant elements of strongly coupled QCD such as confinement, degeneracy of topological sectors, nontrivial $\theta$ dependence, etc.
There is no phase transition in passage from weakly coupled “deformed QCD” to strongly coupled real QCD.

The ground state in “deformed QCD” is saturated by the fractionally charged weakly interacting pseudo-particles (monopoles) which live in 3D.

For inpatient listeners: I formulate the basic results here —
1) The gauge systems usually exhibit the excited “metastable vacuum states” corresponding \( \theta = 2\pi/N \). Their existence is the key element in restoring of the \( 2\pi \) periodicity for the entire system.

2) These “metastable states” \( \theta = 2\pi/N \) can be identified with \( \theta_{\text{ind}}(\vec{x}, t) \), while \( \mu_5 \) can be identified with the tunnelling rate \( |\theta = 2\pi/N\rangle \rightarrow |\theta = 0\rangle \) formulated in terms of Euclidean configurations.
4. Deformed QCD. Basics.

An extra term is put into the Lagrangian in order to prevent the center symmetry breaking

\[
S_{YM}^{\text{M}} = \int_{\mathbb{R}^3 \times S^1} d^4x \left( \frac{1}{2g^2} \text{tr} \left[ F_{\mu\nu}^2(x) \right] + i\theta \int d^4x \frac{1}{16\pi^2} \text{tr} \left[ F_{\mu\nu} \tilde{F}^{\mu\nu} \right] \right),
\]

\[
\Delta S \equiv \int_{\mathbb{R}^3} d^3x \frac{1}{L^3} P \left[ \Omega(x) \right], \quad \Omega(x) \equiv P \left[ e^{i \int dx_4 A_4(x,x_4)} \right]. \quad < A_4^a > = \frac{2\pi \mu^a}{NL}
\]

Parameter “L” is the length of the compactified dimension which is assumed to be small, \(< A_4^a >\) plays the role of the Higgs field in the model.

The infrared description of the theory is a dilute gas of \(N\) types of monopoles, characterized by their magnetic charges, which are proportional to the roots \(\alpha_a \in \Delta_{\text{aff}}\) (Unsal and Yaffe, 2008).
The dual sine-Gordon Lagrangian has the form

\[ S_{\text{dual}} = \int_{\mathbb{R}^3} d^3x \frac{1}{2L} \left( \frac{g}{2\pi} \right)^2 (\nabla \sigma)^2 - \zeta \int_{\mathbb{R}^3} d^3x \sum_{a=1}^{N} \cos \left( \alpha_a \cdot \sigma + \frac{\theta}{N} \right) \]

The dimensional parameter which governs the dynamics of the system is the Debye correlation length of the monopole's gas

\[ m_{\sigma}^2 \equiv L \zeta \left( \frac{4\pi}{g} \right)^2. \]

The average number of monopoles in a "Debye volume" is parametrically large which justifies the semiclassical approximation

\[ \mathcal{N} \equiv m_{\sigma}^{-3} \zeta = \left( \frac{g}{4\pi} \right)^3 \frac{1}{\sqrt{L^3 \zeta}} \gg 1, \quad \text{where fugacity } \zeta \sim e^{-1/g^2} \ll 1. \]
5. Metastable states, their classification, and a P-odd physics.

The metastable states are known to emerge in large N QCD (Witten, 1980).

The corresponding states can be explicitly constructed and studied in a weakly coupled “deformed QCD” model (AZ et al, 2014).

The metastable states are classified by the vacuum expectation values of the creation operators for N different monopole’s types

\[ \mathcal{M}_a(x) = e^{i\alpha_a \cdot \sigma(x)}, \quad \alpha_a \in \Delta_{aff} \]

\[ \langle \mathcal{M}_a(x) \rangle = \exp \left[ \pm i \frac{2\pi k}{N} \right]. \]
An explicit construction of these metastable states is obviously a model-dependent result.

However, the fact of their existence is very generic feature of gauge theories. It is related to necessity to restore the $2\pi$ periodicity in $\theta$ while preserving the $\cos(\theta/N)$ dependence at small $\theta \ll \pi$.

The corresponding metastable states $\langle \mathcal{M}(x) \rangle_k$ are characterized by a non-vanishing expectation values of the topological density operator

$$\langle \frac{1}{16\pi^2} \text{tr} \left[ F_{\mu\nu} \tilde{F}^{\mu\nu} \right] \rangle_k = -i \frac{\zeta}{L} \sin \left( \frac{2\pi k}{N} \right), \quad \theta = 0$$

This feature implies the P-violation at $\theta = 0$ but $k \neq 0$. 
6. **Metastable state and its decay**

**These metastable excited states do decay as a result of the tunnelling transitions.**

**One should not confuse these tunnelling transitions with conventional tunnelling events between degenerate, physically identical but topologically distinct winding \( |k\rangle \) states.**

**Our transitions occur between physically different quantum states \( \langle \mathcal{M}(x) \rangle_k \) with \( \Delta E \neq 0 \).**

**The difference between these two cases can be explained in terms of CM analogy. First case (\( |k\rangle \) - transitions) corresponds to formation of Bloch states while the second case corresponds to the transitions between different Brillouin zones.**
The corresponding tunnelling transitions can be explicitly computed in the “deformed QCD” model in terms of the semiclassical domain wall solutions interpolating between different states.

The result can be represented in terms of the transition rate:
\[
\frac{\Gamma}{V} \sim \exp \{ -N (aN^b) \}, \quad b \approx 3.4, \quad N \equiv m_\sigma^{-3} \zeta = \left( \frac{g}{4\pi} \right)^3 \frac{1}{\sqrt{L^3 \zeta}} \gg 1,
\]

In strongly coupled QCD we cannot compute the rate, but we expect a similar suppression with \( N \).

The required “long range order” can be identified with this slow decay rate from metastable state with effectively non vanishing \( \left\langle \frac{1}{16\pi^2} \text{tr} \left[ F_{\mu\nu} \tilde{F}^{\mu\nu} \right] \right\rangle \neq 0 \)

\[
\mu_5 = \dot{\theta} \sim \frac{\Gamma}{V} \sim \Lambda_{\text{QCD}} \cdot \exp(-N^b) \ll \Lambda_{\text{QCD}}
\]
The tunnelling processes in vacuum do not lead to any emission or absorption of real particles, similar to the multiple tunnelling events in Bloch’s case in condensed matter physics.

These typical tunnelling events simply select an appropriate state $\langle \mathcal{M}(x) \rangle_k$ of the system which is a specific superposition of $|n\rangle$ states.

The ground state corresponds to one specific superposition with $|\theta = 0, k = 0\rangle$. However, another superposition of the same $|n\rangle$ states may form a metastable state $\langle \mathcal{M}(x) \rangle_k$ with $\langle \frac{1}{16\pi^2} \text{tr} \left[ F_{\mu\nu} \tilde{F}^{\mu\nu} \right] \rangle_k \neq 0$, similar to computations in “deformed QCD” model.
If this happens, the corresponding decay of a metastable state \( \langle M(x) \rangle_k \) to the ground state will generate an effective

\[
\mu_5 = \dot{\theta} \sim \Gamma/V \sim \Lambda_{QCD} \cdot \exp(-N^b) \ll \Lambda_{QCD}
\]

The corresponding effective \( \mu_5 \) will be automatically coherent on entire 4d space-time, in the region where a metastable state \( \langle M(x) \rangle_k \) is formed.

This correlation length “\( L \)” is formulated in terms of the Euclidean configurations. It should not be confused with conventional CM correlation length in Minkowski space-time.
We assume that the Euclidean correlation length “l” is the same as a size “L” of a nuclei which is the largest possible scale in the problem.

With this assumption the Euclidean coherent correlation length must scale as

\[ \mu_5 = \dot{\theta} \sim \Gamma/V \sim \Lambda_{\text{QCD}} \cdot \exp(-N^b) \sim L^{-1}, \quad L \sim 10 \text{ fm} \]

A tunnelling transition from a metastable state to the ground state must exhibit a coherent \( \mathcal{P} \) violation in the system on a scale of order \( \sim L \).

This is because a metastable state \( \langle \mathcal{M}(x) \rangle_k \) carries a non-vanishing topological density \( \langle \frac{1}{16\pi^2} \text{tr} [F_{\mu\nu} \tilde{F}^{\mu\nu}] \rangle_k \neq 0 \) which must decay during time \( \tau \sim L/c \).
8. Comparison with Observations

Unfortunately, we can not use the “deformed QCD” model for quantitative analysis in QCD.

However, there are few consequences of this picture which are likely model-independent (not sensitive to any specific details of the framework):

a). For finite system size L the P-odd correlations show the Casimir like scaling $L^{-1}$ rather than naively expected exponential suppression $\exp(-L)$ when a mass gap is present in the system.

b). Some suppression of the measured correlations with increasing size of the system L apparently indeed have been observed. The effect for Au+Au with A=197 is suppressed in comparison with Cu+Cu with A=64.
The correlations demonstrate the universal behaviour similar to the universality observed in all high energy collisions.

We interpret the difference between Au+Au and Cu+Cu as manifestation of a coherent $L^{-1}$ scaling.
d). We expect the same tendency to continue for the LHC energies. Published results for lead (Pb + Pb collisions from ALICE at 2.76 TeV) support this expectation. In fact, such a prediction had been made before the ALICE results were posted on arXiv.
One could argue that this picture must be quite universal because all effects are originated from a metastable vacuum state, not from the colliding particles.

Specifically, there should be no dependence on energy nor charge of colliding ions, similar to the universality of the Hagedorn spectrum in all high energy collisions.

Apparently, the recent LHC results (energy 2.76 TeV in contrast with 200 GeV and 62 GeV at RHIC) support this prediction of this framework.
The total contribution to the energy associated with these soft fluctuations is

\[ e \cdot \kappa \cdot \frac{f(\gamma)}{\Lambda_{QCD}} \sim 5 \cdot 10^{-4}, \text{ (which is about right)} \]

\[ L^2 E_z^{ind} = - \left( \frac{e \theta}{2\pi} \right) l, \text{ where } l = \frac{e}{2\pi} \int d^2 x_{\perp} B_z^{ext} \]

**Evaluation of** \( f(\gamma) \) **is a hard problem. It requires model dependent assumptions, while items a)-d) formulated above are model -independent consequences which are based exclusively on key principles of this framework.**

**What could be the possible technical tools for quantitative computations in strongly coupled QCD?** Lattice simulations? Holographic description?
**Conclusion**

We interpret a relatively large observed intensities of the asymmetries in heavy ion collisions as a coherent vacuum phenomenon when a measured asymmetry is accumulated on large scales $L \sim 10 \text{ fm}$, rather than a result of a conventional small fluctuation with $\sim 1 \text{ fm}$ scale.

This coherent process cannot be expressed in terms of any physical local propagating degrees of freedom. It emerges as a result of a (nonlocal) tunnelling $P$-violating transition between a metastable state $\langle M(x) \rangle_k$ and the ground state when effective coherent $\mu_5$ is inevitably generated.

$$\mu_5 = \dot{\theta} \sim \frac{\Gamma}{V} \sim \Lambda_{QCD} \cdot \exp(-N^b) \sim L^{-1}, \quad L \sim 10 \text{ fm}$$
This picture is drastically different from conventional sphaleron tunnelling. Metastable state with nonzero $\langle M(x) \rangle_k$ emerges as a result of infinite number of tunnelling events between winding states $|n\rangle$. Metastable state is an eigenstate of large gauge transformation operator, while winding state $|n\rangle$ is not.

Proposal: Instead of theoretical speculations I suggest to conduct a real tabletop experiment to study these fundamentally new tunnelling effects between distinct topological sectors.

When the Maxwell $U(1)$ system is formulated on a torus, there will be an extra contribution to the partition function $Z_{\text{top}}(\theta)$ which cannot be expressed in terms of conventional EM-photons (this is because of a nontrivial holonomy of the torus).