Magnetohydrodynamics simulations, flow, flow fluctuations, and vortices

Ajit M. Srivasta
Institute of Physics
Bhubaneswar, India

Collaborators:
Arpan Das, Shreyansh S. Dave, Saumia P.S.
Outline:

1. Magnetic field in relativistic heavy-ion collision experiments. We discuss interdependence of evolution of magnetic field, flow, and flow fluctuations.

2. Enhancement of elliptic flow due to magnetic field, but very complex interdependence. (different results in literature ?)
   Important possibility: possibility of larger value of $\eta/s$ ?, (not sQGP, or no need to cross AdS/CFT bound ?)

3. Relativistic Ideal Magnetohydrodynamics (RMHD) simulations:
   Confirmation of $v_2$ enhancement (but very complex),
   Temporary, localized, increase of magnetic field
   Qualitative patterns in power spectrum if flow fluctuations
   Deformed nuclei: preliminary results for uranium
     - Anomalous elliptic flow,
     - Quadrupole magnetic field (focusing effects?)

4. New possibilities at FAIR, NICA? Vortex (superfluid phase,..) Dynamo effect for magnetic field?

5. Conclusions: Important to find clean signals for magnetic field
Magnetic field in heavy-ion collisions:

Important physics to look for: Chiral magnetic effect, chiral Vortical effect, .......

Magnetic field affects every aspect of flow: enhancement of $v_2$

Most important: Must find clean signal for the presence of initial magnetic field. Qualitative patterns important due to incomplete understanding of magnetic field evolution during very early stages of evolution.

We start with discussion of effect of Magnetic Field on $v_2$:

We earlier pointed out that initial magnetic field can enhance elliptic flow (Mohapatra, Saumia, AMS, MPLA 26, 2477 (2011)). Similar enhancement also found by Tuchin: J.Phys.G39, 025010(2012)

Recent simulations: no effect found: Inghirami etal: arXiv:1609.03042

Important to understand these discrepancies

Will show results of our present simulations
Complex factors affect elliptic flow (in fact, every aspect of flow) in the presence of magnetic field:

Briefly recall basic physics of our original argument:

In presence of magnetic field, there are different types of waves in the plasma. Fast magnetosonic waves: Generalised sound waves with significant contributions from the magnetic pressure.

Basically, distortions of magnetic field in transverse direction costs energy, equation of state stiffer in that direction:

Expect larger sound speed in transverse direction.

Flow velocity proportional to $c_s^2$, so we argued: Flow in x direction will be enhanced, while in y direction will not change:

Conclusion was: $B$ increases $v_2$. 
However, the physics of this is not that simple. Other factors can be present.

For example, it is known that under certain situations, expansion of a conducting plasma into regions of magnetic field gets hindered.

One can expect it from Lenz’s law: expanding conductor squeezes magnetic flux, which should oppose expansion of plasma (cause of squeezing). Such an argument will imply suppression of $v_2$ due to $B$. This will be expected when magnetic field extends well beyond plasma region.

However, this is also not correct, as this completely misses the factor of distortion of magnetic field costing energy (which was the argument we used in our paper arguing for increase of $v_2$.) We can expect that to hold true when magnetic field is entirely contained inside plasma region.

In general, all such factors are present. As we will see later, in some situation one factor will dominate, while in another, the other factor. Along with these two factors, fluctuations also play important role. Final effect is a combination of all these factors.
First we note: very complex flow patterns can develop due to magnetic field when fluctuations are also present.

The expression for group velocity in relativistic MHD

$$v_{gr} = v_p n + t \frac{\sigma \pm 2\delta(a \mp (1 + \delta \cos^2 \theta)) \sin \theta \cos \theta}{2(1 + \delta \cos^2 \theta \pm a)a}$$

$$v_{ph} = n \frac{[(\frac{\rho}{3\omega})c_s^2+vA^2]^{1/2}}{2} (1 + \delta \cos^2 \theta \pm a)^{1/2}$$

$$n = k/k, \ t = [(\frac{B}{B}) \times n] \times n$$

$$a^2 = (1 + \delta \cos^2 \theta) - \sigma \cos^2 \theta, \delta = \frac{c_s^2v_A^2}{(\frac{\rho}{3\omega})c_s^2+vA^2}, \sigma = \frac{4c_s^2v_A^2}{[(\frac{\rho}{3\omega})c_s^2+vA^2]^2}, \omega = (\frac{4\rho}{3}) + B^2$$

Note: Direction of group velocity depends on the coefficient of $t$ above, which depends on local pressure (for a given $B$). Thus: with pressure variations, direction keeps changing: **Very complex flow pattern** (May lead to generation of vortices due to strong fluctuations).
We do not discuss the issue of survival of magnetic field in the plasma. Due to conductivity (Tuchin) magnetic field does not decay very rapidly in the plasma, field diffusion time at least several fm.

We take an initial value of the field, at a given time after the collision, calculated by taking uniformly charged nuclei (spherical or ellipsoidal for deformed case), and Lorentz transforming for oppositely moving nuclei with required impact parameter.

We carry out 3+1 dimensional simulation using Glauber-like initial conditions for QGP, with profile in z-direction being Woods-Saxon with appropriate size. We work in the limit of infinite conductivity: so use equations of Ideal Relativistic MHD:

We follow formalism from:
We first give a brief summary of the formalism:
Conservation of total energy-momentum tensor (perfect fluid QGP + magnetic field):

\[ \partial_\alpha [(\rho + p_g + |b|^2)u^\alpha u^\beta - b^\alpha b^\beta + (p_g + \frac{|b|^2}{2})\eta^{\alpha\beta}] = 0 \]

Maxwell’s equations:

\[ \partial_\alpha (u^\alpha b^\beta - b^\alpha u^\beta) = 0 \]

Where:

\[ b^\alpha = \gamma [\vec{v} \cdot \vec{B}, \frac{\vec{B}}{\gamma^2} + \vec{v}(\vec{v} \cdot \vec{B})] \]

and:

\[ u^\alpha b_\alpha = 0, \text{ and } |b|^2 \equiv b^\alpha b_\alpha = \frac{\vec{B}^2}{\gamma^2} + (\vec{v} \cdot \vec{B})^2 \]
For simulation, these equations are cast in the following form

\[ \frac{\partial U}{\partial t} + \sum_k \frac{\partial F^k}{\partial x^k} = 0 \]

Where different quantities are defined as:

\[ U = (m_x, m_y, m_z, B_x, B_y, B_z, E) \]

\[ m_k = [\rho h \gamma^2 + |\vec{B}|^2]v_k - (\vec{v} \cdot \vec{B})B_k \]

\[ E = \rho h \gamma^2 - p_g + \frac{|\vec{B}|^2}{2} + \frac{v^2|\vec{B}|^2 - (\vec{v} \cdot \vec{B})^2}{2} \]

\[ p = p_g + \frac{|b|^2}{2} \]

\( (F^y, z) \) are similarly defined by appropriate change of indices.

Note: From \( U \) at each stage, independent variables have to be extracted.
$(p_g, \vec{v}, \vec{B})$ are extracted by defining:

\[ S = m_0 \vec{B} \quad W = \rho h \gamma^2 \]

And writing

\[ E = W - p_g + (1 - \frac{1}{2\gamma^2})|\vec{B}|^2 - \frac{S^2}{2W^2} \]

\[ |m|^2 = (W + |\vec{B}|^2)^2(1 - \frac{1}{\gamma^2}) - \frac{S^2}{W^2}(2W + |\vec{B}|^2) \]

These equations are written eventually as a single equation for one unknown $W$ (by rewriting equation for $|m|^2$):

\[ \gamma = \left(1 - \frac{S^2(2W + |\vec{B}|^2) + |m|^2W^2}{(W + |\vec{B}|^2)^2W^2}\right)^{-1/2} \]

\[ f(W) \equiv W - p_g + (1 - \frac{1}{2\gamma^2})|\vec{B}|^2 - \frac{S^2}{2W^2} - E = 0 \]

This equation is solved using Newton-Raphson method to get $W$, from which other independent variables are obtained using above equations.
Limitations of the simulation:

Due to computer limitations, we use small lattice (200x200x200) so small nuclei used (copper), also for small times only up to maximum of 3 fm time, sometime much shorter time. We use smaller energy CMS energy of 20 GeV, for very large energies magnetic field becomes very large near receding nuclei (it is 3+1 dimensional simulation), causing problem with fluctuations difficult to run for long times.

General problem when magnetic field energy density becomes much larger than the plasma density. Same problem was found in other simulation also (Inghrami et al. where no effect on $v_2$ was found)

For elliptic flow, we have studied details of the dependence of elliptic flow on magnetic field, and it seems to crucially depend on the relative profiles of $B$ and plasma density.

We first present these results
Elliptic flow in the presence of magnetic field:

We see that magnetic field enhances $v_2$, but only up to impact parameter of about 6 fm, after that it suppresses it. Also enhancement peaks for small impact parameter. Why?
Note: Magnetic field almost monotonically increases with impact parameter.
Magnetic field plots (top) and plasma density plots (bottom) for small 1 fm (left) and large 7 fm (right) impact parameter.

Left: B contained entirely within plasma region, expect $v_2$ enhancement from anisotropic sound speed.

Right: B extends well outside plasma region, expect $v_2$ suppression from flux squeezing.
Result-2: Temporary increase of magnetic field due to flux-rearrangement by evolving plasma density fluctuations. Evolving fluctuations can push around flux lines, leading to temporary, localized concentration of flux.

Important for chiral magnetic effect which is sensitive to local magnetic field (instanton size regions). We could only study small fluctuations, for large fluctuations effect can be stronger.
\textbf{Result-3: Effect on power spectrum}

Top: small magnetic field: $0.1 - 0.4 \, \text{mpi}^2$. Very tiny effect

Strong magnetic field $5 \, \text{mpi}^2$. Significant effect, note first few flow coefficients show some even-odd power difference.

Here magnetic field put in by hand (also taken constant along Y direction for stable simulation (for Gauss’ law))
To show the fluctuations mask this signal we show power spectrum in the presence of small magnetic field ($1 \text{ mpi}^2$), but in the absence of any initial state fluctuations. Note: even-odd effect still very strong.

Much stronger magnetic field: $15 \text{ mpi}^2$. Very clear even-odd power difference. Qualitative in nature. Arises from reflection symmetry about the axis of magnetic field, so clear effect only when B dominates over random fluctuations.

(important implications for low $l$ modes of CMBR power spectrum)

To show the fluctuations mask this signal we show power spectrum in the presence of small magnetic field ($1 \text{ mpi}^2$), but in the absence of any initial state fluctuations. Note: even-odd effect still very strong.
**Result-4: New possibilities with deformed nucleus**

Following configurations will lead to anomalous elliptic flow:

(a) Elliptical plasma region, but magnetic field induces larger flow along semi-minor axis of the ellipse, this suppresses $v_2$. For strong magnetic field case it can lead to negative elliptic flow.

(b) Isotropic QGP region but B gives rise to non-zero elliptic flow.

We represent these situations by producing QGP region and magnetic field independently using suitable impact parameters, and then combine the two profiles as per our choice.
Suppression of $v_2$ due to magnetic field pointing along x axis (semi-minor axis)
Isotropic QGP region, but non-zero $v_2$ due to $B$, which increases monotonically with $B$. 

$\sqrt{s} = 20$ GeV  
$B_{\text{time}} = 0.4$ fm
A very interesting possibility: Crossed-configuration for deformed (Uranium) nuclei. Produced magnetic field is quadrupolar.

Expect phenomena arising from beam focusing (along longitudinal direction), along with very large $v_4$. Under investigation at present:
Dynamo effect?

Superfluid vortices at FAIR and NICA may be possible: (Arpan Das, Shreyansh S. Dave, AMS: arXiv: 1607.00480)

This can lead to remarkable possibilities:

Vortices (turbulence) are known to strongly increase magnetic field, the so-called Dynamo effect.

We studied evolution of magnetic field in the presence of vortex configuration (as per our above paper).

We see strong increase in the magnetic field in the presence of vortices.

Note: Dynamo effect for ideal MHD may be possible in strong flux-folding regime as expected in the presence of vortices.
Central magnetic field evolution in presence of vortices (Cu-Cu, impact parameter 2 fm).
Conclusions:

Very important to study evolution of magnetic field in the presence of fluctuations. Can give clean signals of the presence of magnetic field.

Fluctuations get strongly affected (elliptic flow, Power spectrum) due to magnetic field.

At the same time, fluctuations can feed into magnetic field, increasing it temporarily. For dramatic example of vortex induced turbulence, one may get dynamo like effect (exciting possibilities for FAIR and NICA).

Deformed nuclei open up new possibilities: Anomalous elliptic flow, quadrupolar magnetic field may lead to beam focusing effects, suppressing transverse expansion (an also deviation from Bjorken scaling) in the longitudinal direction.
Thank You