Photon and Di-lepton Emissions from Axially Charged Plasma

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Based on

“Spin Polarized Photons and Di-leptons from Axially Charged Plasma”


Kiminad Mamo and HUY
Axial Charge is P- and CP-odd

\[ \psi_L \rightarrow q_L \text{ (quark), } \bar{q}_R \text{ (anti-quark)} \]
\[ \psi_R \rightarrow q_R \text{ (quark), } \bar{q}_L \text{ (anti-quark)} \]

\[ P : q_L \leftrightarrow q_R, \; \bar{q}_L \leftrightarrow \bar{q}_R \]
\[ C : q_L \leftrightarrow \bar{q}_L, \; q_R \leftrightarrow \bar{q}_R \]

Axial Charge

\[ J_A^0 = N(q_L) + N(\bar{q}_L) - N(q_R) - N(\bar{q}_R) \]
Other examples of P- and CP-odd quantities include

- Topological charge $\vec{E} \cdot \vec{B} \sim \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$
- $\theta$ angles
- Chiral Magnetic Conductivity $\sigma_\chi$ in $\vec{J}_V = \sigma_\chi \vec{B}$

On the other hand

Chiral Vortical Effect $\vec{J} = \sigma_\chi \vec{\omega}$ is P-odd, but CP-even

Doesn’t need the presence of axial charge in general
Local P- and CP-odd domain in heavy-ion collisions is a key to the observable phenomena arising from triangle anomaly (Kharzeev-McLerran-Warringa).

Eg. Charge separation by Chiral Magnetic Effect

Typical Problem

There are other background effects to the proposed observables since these observables are not P- and CP-odd.
P- and CP-odd Observables

- Using discrete symmetries, P and C
  
  Clean Isolation: No (easy) background effects

- Unambiguous signal of axial charge and triangle anomaly

- Drawback: Need to be event-by-event since QCD is a P- and CP-even theory
Question:
What are the possible P- and CP-odd observables in photon and di-lepton emissions?
Photons

Spin polarization (helicity $h$) is P-odd

$$\epsilon^\mu_{\pm} = \frac{1}{\sqrt{2}} (0, 1, \pm i, 0), \quad \vec{k} = k \hat{x}^3$$
Photons are C-eigenstates, hence the emission rates are C-even quantities

**P- and CP-odd Observable**

\[ A_{\pm \gamma} \equiv \frac{d\Gamma}{d^3k}(\epsilon_+) - \frac{d\Gamma}{d^3k}(\epsilon_-) \]

\[ - \frac{d\Gamma}{d^3k}(\epsilon_-) + \frac{d\Gamma}{d^3k}(\epsilon_-) \]
Di-leptons

\[ \Gamma^{s_1, s_2} = \frac{d\Gamma^{s_1, s_2}}{d^3 p_1 d^3 p_2} \]

where \((s_1, s_2) = (\pm 1/2, \pm 1/2)\) are spin alignments of the di-lepton pair along the momentum direction.
\[ P: \ e^-_L \leftrightarrow e^-_R, \ e^+_L \leftrightarrow e^+_R \]
\[ C: \ e^-_L \leftrightarrow e^+_L, \ e^-_R \leftrightarrow e^+_R \]

which gives

\[ P: \ \Gamma^{+\frac{1}{2},+\frac{1}{2}} \leftrightarrow \Gamma^{-\frac{1}{2},-\frac{1}{2}}, \ \Gamma^{+\frac{1}{2},-\frac{1}{2}} \leftrightarrow \Gamma^{-\frac{1}{2},+\frac{1}{2}} \]
\[ C: \ \Gamma^{\pm\frac{1}{2},\pm\frac{1}{2}} \leftrightarrow \Gamma^{\pm\frac{1}{2},\pm\frac{1}{2}}, \ \Gamma^{+\frac{1}{2},-\frac{1}{2}} \leftrightarrow \Gamma^{-\frac{1}{2},+\frac{1}{2}} \]

**P- and CP-odd Observable**

\[ A_{\pm\bar{\nu}} \equiv \frac{\Gamma^{+\frac{1}{2},+\frac{1}{2}} - \Gamma^{-\frac{1}{2},-\frac{1}{2}}}{\Gamma^{+\frac{1}{2},+\frac{1}{2}} + \Gamma^{-\frac{1}{2},-\frac{1}{2}}} \]
No Such Observables for Massless Leptons

In fact, $A_{\pm l} \sim (\Gamma^{+\frac{1}{2},+\frac{1}{2}} - \Gamma^{-\frac{1}{2},-\frac{1}{2}}) \propto m^2$
How to Compute $A_{\pm \gamma}$ and $A_{\pm \bar{l}}$?

Photon and di-lepton emissions are computed from the current-current Wightmann functions $G_{\mu \nu}^{<}$.

They probe P- and CP-odd properties of the plasma.
Photon Emission with Polarization $\epsilon^{\mu}$

\[
\frac{d\Gamma}{d^{3}k}(\epsilon^{\mu}) = \frac{e^{2}}{(2\pi)^{3}2|k|} (\epsilon^{\mu})^{\ast} \epsilon^{\nu} G^{<}_{\mu\nu}(k) \bigg|_{k^{0}=|k|}
\]

Using a thermal relation

\[
(\epsilon^{\mu})^{\ast} \epsilon^{\nu} G^{<}_{\mu\nu}(k) = \frac{-2}{e^{\beta k^{0}} - 1} \text{Im} \left[ (\epsilon^{\mu})^{\ast} \epsilon^{\nu} G^{R}_{\mu\nu}(k) \right]
\]

we have

\[
\frac{d\Gamma}{d^{3}k}(\epsilon^{\mu}) = \frac{e^{2}}{(2\pi)^{3}2|k|} \frac{-2}{e^{\beta|k|} - 1} \text{Im} \left[ (\epsilon^{\mu})^{\ast} \epsilon^{\nu} G^{R}_{\mu\nu} \right] k^{0}=|k|
\]
Rotational invariance dictates

\[ G^R_{ij}(k) = A(k)\delta_{ij} + B(k)k_i k_j + i\sigma_\chi(k)\epsilon_{ijk}k^k \]

where the last is a P- and CP-odd term with chiral magnetic conductivity \( \sigma_\chi(k) \)

by Kharzeev-Warringa.

It gives rise to a frequency-momentum dependent chiral magnetic effect

\[ \vec{J}(k) = \sigma_\chi(k)\vec{B}(k) \]

This implies when \( \vec{k} = k\hat{z} \)

\[ G^R_{11} = G^R_{22}, \quad G^R_{12} = -G^R_{21} = i\sigma_\chi(k)k \]
Using these relations to compute

\[
\frac{d\Gamma}{d^3\vec{k}}(\epsilon^\mu) = \frac{e^2}{(2\pi)^3 2|\vec{k}|} \frac{-2}{e^\beta|\vec{k}| - 1} \text{Im} \left[ (\epsilon^\mu)^* \epsilon^\nu G_{\mu\nu}^R \right] k^0 = |\vec{k}|
\]

we have

\[
A_{\pm\gamma} = \frac{\text{Im} G_+^R - \text{Im} G_-^R}{\text{Im} G_+^R + \text{Im} G_-^R} = \frac{2 \text{Re} G_{12}^R}{\text{Im} \text{Tr} G^R}
\]

where \( G_\pm^R \equiv G_{11}^R \pm iG_{12}^R \)

Recalling \( G_{12}^R = i\sigma^\chi(k)k \) we see that

\( A_{\pm\gamma} \) measures the imaginary part of chiral magnetic conductivity at light-like momenta \( k^0 = |\vec{k}| \)
Di-lepton Emission with Spin Alignments

\[ \Gamma^{s_1,s_2} = \frac{d\Gamma^{s_1,s_2}}{d^3p_1 d^3p_2} \]

where \((s_1, s_2) = (\pm 1/2, \pm 1/2)\) are spin alignments of the di-lepton pair along the momentum direction.
Di-lepton Emission with Spin Alignments

\[
\frac{d\Gamma^{s_1, s_2}}{d^3 p_1 d^3 p_2} = \frac{e^2 e_i^2}{(2\pi)^6} \left( \frac{1}{p_f^2} \right)^2 \frac{1}{2E_{\vec{p}_1}} \frac{1}{2E_{\vec{p}_2}} G^<_{\mu\nu}(p_f) \times (-1) \left[ \bar{v}(\vec{p}_2, s_2) \gamma^\mu u(\vec{p}_1, s_1) \right] \left[ \bar{u}(\vec{p}_1, s_1) \gamma^\nu v(\vec{p}_2, s_2) \right],
\]

Using the same relations, we arrive at

\[
A_{\pm i\bar{i}} = \left( \frac{2 \cos \theta}{1 + \cos^2 \theta} \right) \cdot \left. \frac{\text{Im } G_+^R - \text{Im } G_-^R}{\text{Im } G_+^R + \text{Im } G_-^R} \right|_{k=k_1+k_2},
\]

\(A_{\pm i\bar{i}}\) also measures the imaginary part of chiral magnetic conductivity
Model Computations

- Strong Coupling (AdS/CFT correspondence)
- Weak Coupling (Planned)
AdS/CFT Model Computation

Chiral anomaly is represented by a 5D Chern-Simons term

$$\epsilon^{MNPQR} A_M F_{NP} F_{QR} \supset A_0 (\partial_r A_1) (\partial_3 A_2)$$

$$A_0 \sim \mu_A \text{ and } \partial_3 \sim k$$

It gives rise to $G_{12}^R \sim \sigma_{\chi}(k)$
The effect is about a percent level.
Thank You for Listening