Electromagnetic fields in relativistic heavy-ion collisions

Kirill Tuchin

QCD Workshop on Chirality, Vorticity and Magnetic Field in Heavy Ion Collisions

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• Electromagnetic field in relativistic heavy-ion collisions at finite electrical conductivity.

• Effect of finite chiral conductivity on field evolution.

• Some phenomenological effects of B: flow, bound states, photons.
HEAVY ION COLLISIONS: TIME EVOLUTION

valence electric charges

Plasma created

Ions about to collide

Ion collision

Quarks, gluons freed

Plasma decays

Number of valence quarks at y=0 decreases with energy: baryon stopping.

In the following I set dN_{val}/dy=0

\[
\frac{dN_{\text{val}}}{dy} \sim e^{-\Delta R (Y-y)} + e^{-\Delta R (Y+y)}, \quad \Delta_R \approx 0.47
\]
ELECTROMAGNETIC FIELD IN HEAVY ION COLLISIONS

Transverse plane

Valence quarks

Valence quarks

B

Ze

B ~ Ze \frac{b}{R^3} \gamma

Z_{Au} = 79, b\sim R = 7 \text{ fm}, \gamma = 100 \Rightarrow

eB = (200 \text{ MeV})^2 \approx m_{\pi}^2

B \sim 10^{18} \text{ G}

Reaction plane

Impact parameter b

Valence quarks

Valence quarks

Impact parameter b
Consider first a single point charge $e$ moving with velocity $v$:

\[ \nabla \cdot B = 0, \quad \nabla \times E = -\frac{\partial B}{\partial t}, \]
\[ \nabla \cdot D = e\delta(z - vt)\delta(b), \quad \nabla \times H = \frac{\partial D}{\partial t} + \sigma E + ev\hat{z}\delta(z - vt)\delta(b) \]

In momentum space:

\[ H_{\omega k} = -2\pi iev\frac{k \times \hat{z}}{\omega^2 \tilde{\epsilon} \mu - k^2} \delta(\omega - k_z v), \quad E_{\omega k} = -2\pi i\frac{\omega \mu v \hat{z} - k / \epsilon}{\omega^2 \tilde{\epsilon} \mu - k^2} \delta(\omega - k_z v) \]

where $\tilde{\epsilon} = \epsilon + i\sigma / \omega$

Time dependence of electromagnetic field is determined by singularities in the complex $\omega$-plane with finite imaginary part. Take for simplicity $\epsilon = \mu = 1$ (neglect the polarization and magnetization response of QGP).
**SOLUTION TO MAXWELL EQUATIONS**

\[ B(t, r) = \frac{e}{2\pi\sigma} \hat{\phi} \int_0^\infty \frac{J_1(k_\perp b) k_\perp^2}{\sqrt{1 + \frac{4k_\perp^2}{\gamma^2\sigma^2}}} \exp \left\{ \frac{1}{2} \sigma \gamma^2 x_- \left( 1 - \sqrt{1 + \frac{4k_\perp^2}{\gamma^2\sigma^2}} \right) \right\} dk_\perp, \]

where \( x_- = t - z/v \)

Since \( k_\perp \sim 1/b \), the relevant parameter is \( \lambda = \gamma \sigma b \)

If \( \lambda \ll 1 \):

\[ E = \frac{e\gamma}{4\pi} \frac{b - vx_- \hat{z}}{\left( b^2 + \gamma^2 v^2 x_-^2 \right)^{3/2}}, \quad B = \frac{e\gamma}{4\pi} \frac{vb\hat{\phi}}{\left( b^2 + \gamma^2 v^2 x_-^2 \right)^{3/2}}, \]

No plasma effect on field!

At late times: \( E, B \sim \frac{1}{x_-^3} \)  This behavior sets in at \( x_- = \frac{b}{\gamma} \sim 0.1 \text{ fm/c} \)

Plasma is created at \( t=0.5-1 \text{ fm/c} \)
If $\lambda \gg 1$: 

$E_r = B_\phi = \frac{e}{2\pi} \frac{b\sigma}{4x_-^2} e^{-\frac{b^2 \sigma}{4x_-}},$ 

$E_z = -\frac{e}{4\pi} \frac{x_- - b^2 \sigma/4}{\gamma^2 x_-^3} e^{-\frac{b^2 \sigma}{4x_-}}$

At late times: $E, B \sim \frac{1}{x_-^2}$

This behavior sets in only at $x_- = \frac{b^2 \sigma}{4} = \frac{b\lambda}{4\gamma} \gg \frac{b}{\gamma}$

i.e. at much later time.

For $\gamma=100$, $\sigma \approx 5.8$ MeV, $b=7$ fm: $\lambda=19 \Rightarrow$ RHIC and LHC are certainly in the “diffusion regime”

**Field of a single charge**

For $\gamma=100$, $\sigma \approx 5.8$ MeV, $b=7$ fm, $z=0$. 

**Total field of two ions**

For $\gamma=100$, $\sigma \approx 5.8$ MeV, $b=7$ fm, $z=0$. 

$eB_y/m^2_\pi \approx 5.8$ MeV, exact “diffusion”
SPATIAL DISTRIBUTION OF E&M FIELD IN PLASMA

\[ \frac{eB_y}{m_{\pi}^2} \]

\[ \frac{eB_x}{m_{\pi}^2} \] \[ \langle eB_x \rangle = 0 \]

\[ \frac{eE_y}{m_{\pi}^2} \]

\[ \frac{eE_x}{m_{\pi}^2} \] \[ \langle eE_y \rangle = 0 \]

\[ \langle eE_x \rangle = 0 \]
Symmetry arguments require vanishing of all components of EM field except $B_y$ only on average. Since distribution of proton positions in nuclei fluctuates, all field components fluctuate as well.
TOPOLOGICAL EFFECTS

Maxwell-Chern-Simons theory: \[ L = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - A_\mu j^\mu - \frac{c}{4} \theta \tilde{F}^{\mu\nu} F_{\mu\nu} \quad c = N_c \sum_f q_f^2 e^2 / 2\pi^2 \]

\[ \partial_\mu F^{\mu\nu} = j^\nu - c \tilde{F}^{\mu\nu} \partial_\mu \theta \]
\[ \partial_\mu \tilde{F}^{\mu\nu} = 0 \]

\[ \nabla \cdot B = 0 , \]
\[ \nabla \cdot E = \rho - c \nabla \theta \cdot B , \]
\[ \nabla \times E = -\partial_t B , \]
\[ \nabla \times B = \partial_t E + j + c(\partial_t \theta B + \nabla \theta \times E) , \]

Neglect \( \nabla \theta \) and assume \( \partial_t \theta = \mu_5 = \text{const.} \), i.e. describe the chiral anomaly by adding current \( j = \sigma_\chi B \) where \[ \sigma_\chi = \mu_5 \frac{e^2}{2\pi^2} N_c \sum_f q_f^2 \] chiral conductivity induced by the QED anomaly

The model \( \partial_\mu \theta = (\mu_5, \mathbf{0}) \) is not realistic, since the typical CP-odd bubble size is \( \sim 1/g^2 T \sim \text{fm} \). But it is a useful starting point because (i) it reproduces CME and (ii) analytically solvable.

Kharzeev et al
STATIC TOPOLOGICAL SOLUTIONS?

Maxwell-Chern-Simons admits non-trivial static solutions, “magnetic nodes”, that satisfy

\[ \nabla \cdot B = 0 , \]
\[ \nabla \times B = \sigma_\chi B \]
\[ \hat{r} \cdot B|_{\partial D} = 0 \]

These solutions have radial sizes \( R \sim 1/\sigma_\chi \sim 200 \text{ fm} \) \( \Rightarrow \) irrelevant for QGP.

More important: are these solutions a model artifact?
Far away from any sources Maxwell equations in momentum space read

\[
\begin{align*}
  k \cdot B_{\omega,k} &= 0, \\
  \epsilon k \cdot E_{\omega,k} &= 0, \\
  k \times E_{\omega,k} &= \omega B_{\omega,k}, \\
  k \times B_{\omega,k} &= -\omega \epsilon E_{\omega,k} - i\sigma_\chi B_{\omega,k}
\end{align*}
\]

Electromagnetic waves have the following dispersion relation:

\[
[\omega(\omega + i\sigma) - k^2]^2 = \sigma_\chi^2 k^2
\]

Four solutions: \( \omega_{\lambda_1,\lambda_2} = -\frac{i\sigma}{2} + \lambda_1 \sqrt{k^2 + \lambda_2 \sigma_\chi k - \sigma^2/4} \) \quad \lambda_1, \lambda_2 = \pm 1

All four poles lie in the lower-half plane of complex \( \omega \) except when \( k < \sigma_\chi \)

In this case \( \text{Im}\omega_{+} > 0 \), indicating a solution exponentially growing with time. \( B \sim e^{-i\omega t} \)

Is this instability connected to the magnetic nodes?
\textbf{FIELD OF A POINT CHARGE}

\[ \nabla \cdot B = 0, \]
\[ \nabla \cdot D = e\delta(z - vt)\delta(b), \]
\[ \nabla \times E = -\partial_t B, \]
\[ \nabla \times H = \partial_t D + \sigma \chi B + ev\dot{z}\delta(z - vt)\delta(b) \]

Magnetic field of the moving charge:
\[
B_{\omega, k} = \frac{(k \times \hat{z})[\omega(\omega + i\sigma) - k^2] - i\sigma \chi k \times (k \times \hat{z})}{[\omega(\omega + i\sigma) - k^2]^2 - \sigma^2 \chi k^2} \left( -2\pi i \right) ev\delta(\omega - k_z v)
\]

Unlike the free case, \( k_z \) is now fixed by the delta-function (equation of motion).

The dispersion relation now obeys
\[
\left( -\frac{\omega^2}{v^2\gamma^2} + i\omega\sigma - k^2 \right)^2 - \sigma^2 \chi \left( \frac{\omega^2}{v^2} + k^2 \right) = 0
\]

Charge trajectory: \( x = t - \frac{z}{v} = 0 \)
Two upper poles:
At $k_\perp \to 0$ $\omega = v^2 \gamma^2 (i \sigma_{\pm} \sigma_{\chi})$; at $k_\perp \to \infty$ $\omega = \pm iv \gamma k_\perp \pm \frac{1}{2} v \gamma \sigma_{\chi} \sqrt{\gamma^2 - 1}$
$\Rightarrow \omega \to \infty$ as $\gamma \to \infty \Rightarrow$ upper poles go away $\Rightarrow$ no field at $x < 0$ (causality!)

Two lower poles:
At $k_\perp \to 0$ $\omega \to 0$
(finite at $\gamma \to \infty$)

FIG. 1: Four solutions of (47) at $\sigma = 5.8$ MeV, $\sigma_{\chi} = 1$ MeV, $\gamma = 100$. Horizontal and vertical axes are in units of GeV. Each line is a unique function of $k_\perp$. Squares, circles and triangles indicate the positions of the poles at $k_\perp = 0.1, 0.6, 1.1$ GeV respectively.

At $k_\perp < \sigma_{\chi}$ one of the lower poles cross the real axis and go into the upper plane $\Rightarrow$ trouble with causality.
DIFFUSION APPROXIMATION

For all practical purposes \( x_- \gg \frac{1}{\sigma v^2 \gamma^2}, \quad x_- \gg \frac{b}{v \gamma} \)

Estimate: \( k_\perp \sim 1/b, \sigma=5.8 \text{ MeV}, \gamma=100 \Rightarrow \frac{1}{\sigma v^2 \gamma^2} \sim 3 \cdot 10^{-3} \text{ fm} \ll \frac{1}{Q_s} \approx 0.2 \text{ fm}. \)
\( b/\gamma=0.1 \text{ fm} < \frac{1}{Q_s} \approx 0.2 \text{ fm}. \)

The dispersion relations

\[
\omega_{1,2} = \frac{-i\sigma k_\perp^2 \pm k_\perp \sigma \chi \sqrt{k_\perp^2 - \sigma^2 - \sigma_\chi^2}}{\sigma^2 + \sigma_\chi^2}
\]

\[
B = \int \frac{d^2 k_\perp}{(2\pi)^2} e^{ik_\perp \cdot b} \left\{ \frac{i}{\omega_2 - \omega_1} \left[ e^{-i\omega_1 x_-} f(\omega_1) \theta(k_\perp - \sigma \chi) - e^{-i\omega_2 x_-} f(\omega_2) \right] \theta(x_-) \right. \\
- \left. \frac{i}{\omega_2 - \omega_1} e^{-i\omega_1 x_-} f(\omega_1) \theta(\sigma \chi - k_\perp) \theta(-x_-) \right \}
\]

acausal term is a manifestation of instability or model artifact?
\[ B_\phi = \frac{eb}{8\pi x_-^2}e^{-\frac{b^2\sigma}{4x_-}} \left[ \sigma \cos \left( \frac{b^2\sigma\chi}{4x_-} \right) + \sigma \chi \sin \left( \frac{b^2\sigma\chi}{4x_-} \right) \right] \]

FIG. 2: Magnetic field of a point charge as a function of time \( t \) at \( z = 0 \). (Free space contribution is not shown). Electrical conductivity \( \sigma = 5.8 \) MeV. Solid line on both panels corresponds to \( B = B_\phi \) at \( \sigma_\chi = 0 \). Broken lines correspond to \( B_\phi \) (dashed), \( B_r \) (dashed-dotted) and \( B_z \) (dotted) with \( \sigma_\chi = 15 \) MeV on the left panel and \( \sigma_\chi = 1.5 \) MeV on the right panel. Note that the vertical scale on the two panels is different.
Applications: 1. Plasma Flow

Azimuthal anisotropy of plasma is generated by the pressure gradients due to the almond shape geometry of the fireball.

Elliptic flow describes the azimuthal momentum space anisotropy of particle emission.

\[
\frac{dN}{d\phi} = C \left( 1 + 2v_2 \cos(2\phi) + 2v_4 \cos(4\phi) + \ldots \right)
\]

In practice one runs a hydro code with unknown viscosity of plasma which is fitted to reproduce \(v_2, v_4\), etc.
Applications: 1. Plasma Flow in Strong B

Viscous pressure tensor in magnetic field is axially symmetric

\[ \Pi_{\alpha\beta} = \sum_{n=0}^{4} \eta_n \, V_{(n)}^{\alpha\beta} \]

gradients of plasma velocity

5 shear (+ 2 bulk) viscosities

In strong field \( \omega_B^{-1} \ll \lambda_{mfp} \ll R_A \) the viscous pressure tensor simplifies:

\[ \Pi^B_{\alpha\beta} = 2\eta_B \begin{pmatrix} \frac{1}{2} V_{xx} & 0 \\ 0 & V_{yy} \end{pmatrix} \]

Asymmetry in the xy-plane!

Compare this with the case \( B=0 \):

\[ \Pi^0_{\alpha\beta} = 2\eta \begin{pmatrix} V_{xx} & V_{xy} \\ V_{yx} & V_{yy} \end{pmatrix} \quad \eta \neq \eta_B \]

Navier-Stokes equations:

\[ \rho \left( \frac{\partial V_\alpha}{\partial t} + V_\beta \frac{\partial V_\alpha}{\partial x_\beta} \right) = -\frac{\partial P}{\partial x_\alpha} + \frac{\partial \Pi_{\alpha\beta}}{\partial x_\beta} \]

At later times give \( V_x/V_y = \sqrt{2} \)

Anisotropy:

\[ \frac{V_x^2 - V_y^2}{V_x^2 + V_y^2} = 1 - \frac{1}{2} = \frac{1}{3} \]

“Elliptic flow” is generated by magnetic field. So the viscosity extracted from the hydro at \( B=0 \) is underestimated.
APPLICATIONS: 2. DISSOCIATION OF J/ψ

J/ψ melts in plasma. Expect its suppression in heavy-ion collisions vs pp ⇒ “smoking gun” of plasma.

However, there are other mechanisms to destroy quarkonium.
APPLICATIONS: 2. DISSOCIATION OF $J/\psi$

Lab frame: center-of-mass frame of a heavy-ion collision

$J/\psi$ rest frame

**APPLICATIONS: 2. DISSOCIATION OF J/ψ**

**J/ψ rest frame.** There is finite quantum probability for the anti-quark (e<0) to tunnel through the potential barrier and go to $y \to -\infty$.

\[-eE_y = |e|E_y\]

\[|e|E_y + \varepsilon_{\text{kin}}\]

\[\varepsilon_b\]

This is $J/\psi \rightarrow D^+D^-$ decay.

Dissociation rate can be calculated in the WKB approximation as a tunneling rate of quark thru the potential barrier.
FIG. 1: Dissociation rate of $J/\psi$ at $eB_0 = 15m_{\pi}^2$, $\phi = \pi/2$ (in the reaction plane), $\eta = 0$ (midrapidity) as a function of (a) $P_{\perp}$ at $\varepsilon_b = 0.16$ GeV and (b) $\varepsilon_b$ at $P_{\perp} = 1$ GeV.
APPLICATIONS: 3. SYNCHROTRON RADIATION

Unexplained excess of Photons in Heavy Ion collisions
Spacing between the Landau levels $\sim eB/\varepsilon$, while their thermal width $\sim T$. When $eB/\varepsilon \gtrsim T$ it is essential to account for quantization of fermion spectra.

Magnetic field does no work, thus energy is conserved. Magnetic Lorentz force has no component along the $B$-direction:

$$\varepsilon_j = \omega + \varepsilon_k, \quad p = q + \omega \cos \theta$$

Angular distribution of the power spectrum:

$$\frac{dI^j}{d\omega d\Omega} = \sum_f \frac{z_f^2 \alpha}{\pi} \omega^2 \sum_{k=0}^j \Gamma_{jk} \{ |M_\perp|^2 + |M_\parallel|^2 \} \delta(\omega - \varepsilon_j + \varepsilon_k)$$

Matrix elements are well-known functions of Laguerre polynomials. Sokolov, Ternov (1968) and others
Peaks correspond to cutoff frequencies. For $j \to k$ transition

$$
\omega \leq \omega_{s,jk} \equiv \frac{m_j - m_k}{\sin \theta} = \frac{\sqrt{m^2 + 2je_f B} - \sqrt{m^2 + 2ke_f B}}{\sin \theta}
$$

FIG. 1: Spectrum of synchrotron radiation by $u$ quarks at $eB = m_\pi^2$, $y = 0$, $\phi = \pi/3$: (a) contribution of 10 lowest Landau levels $j \leq 10$; several cutoff frequencies are indicated; (b) summed over all Landau levels.

Azimuthal asymmetry of photons: experimental data

\[
\frac{dN}{d\phi} = C \left( 1 + \frac{8}{7} \cos(2\phi) + \frac{1}{5} \cos(4\phi) + \ldots \right)
\]

Applications: 3. Synchrotron Radiation

Azimuthal asymmetry of photons in magnetic field

\[ dN = \left( V_2 \right) \text{ GeV}^{-2} \]
SUMMARY AND OUTLOOK

• Electromagnetic field produced in relativistic heavy-ion collisions is, probably, the strongest in nature: $B \sim 10^{18}$ G. It lives long enough to influence the properties of the QGP.

• It induces novel topological effects that are accessible to experiments.

• I discussed several effects that magnetic field has on heavy-ion phenomenology:
  ✴ Reduction of plasma viscosity
  ✴ Dissociation of bound states
  ✴ Synchrotron radiation

• We need a coherent framework that incorporates magnetic field in heavy-ion collision phenomenology.